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FISCAL DEFICITS, CAPITAL CONTROLS, AND THE DUAL EXCHANGE RATE (*)

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Resumen Este artículo analiza las implicaciones de los controles de capitales o de un tipo de cambio dual durante una crisis de balanza de pagos. Ambas medidas provocan una devaluación endógena y anticipada. Se deriva el comportamiento de la economía antes de la devaluación con métodos numéricos y se demuestra que éste depende de manera importante de las preferencias individuales. En concreto, se puede mejorar la balanza por cuenta corriente y puede aumentar la demanda real de dinero antes de la devaluación. El tipo de interés doméstico a corto plazo puede ser superior al tipo extranjero. Además, los controles de capitales no disminuyen necesariamente el bienestar. Finalmente, un gobierno con un horizonte corto puede imponer controles de capitales aunque éstos implican un bienestar inferior.

Abstract This paper analyzes the effect of restrictions on capital movements or of a dual exchange rate during a balance-of-payments crisis. Both lead to an endogenous, anticipated devaluation. The behavior of the economy before the devaluation is derived by using numerical methods and is shown to depend strongly on preferences. In particular, there can be an improving current account deficit and increasing real money holdings before the devaluation. The domestic short term interest rate can be higher than the foreign rate. Moreover, restricted capital mobility does not necessarily decrease welfare. Finally, a government with a short horizon may prefer capital controls even when they decrease welfare.

1. INTRODUCTION

Balance-of-payments crises are phenomena occuring with frequency in several countries. Most economic variables are affected during such a crisis and the actual behavior of the economy is ussually complex. Economists are only starting to understand the various mechanisms at work during a crisis, in particular through a growing literature starting with the seminal work of Krugman (1979). The analysis, however, neglects various institutional factors that strongly affect the dynamics of the balance-of-payments crises. A major factor is the pervasive use of capital account restrictions or of dual exchange rate systems during a crisis. Most of the literature typically assumes capital mobility and shows that large capital outflows force the government to abandon a fixed exchange rate, through losses of foreign exchange reserves.

The effect of restricted capital mobility has been examined by various authors, but several aspects are still not well understood. Previous studies have shown that even a complete prohibition of capital movements cannot avoid the collapse of a fixed exchange rate (1): as households cannot buy foreign assets, there is a spill-over effect to the trade balance. The drain of foreign reserves thus comes from a trade deficit. It was also

(*) This paper draws on a chapter of my Ph. D. thesis. I would like to thank Susan Collins, Jeffrey Sachs, Philippe Weil, and Charles Wyplosz for useful comments. All remaining errors are mine.

(1) See Auernheimer (1987), Bacchetta (1987), and Park and Sachs (1987). The first analysis of the role of capital controls in a balance-of-payments crisis was made by Wyplosz (1986).

shown that restricted capital mobility leads to an *anticipated, endogenous* devaluation: when the government abandons the fixed exchange rate, it stops financing the current account deficit with its foreign exchange reserves. A devaluation must then occur to balance the current account. As both the timing and the size of the devaluation are endogenous, however, the actual behavior of the economy before the devaluation could not be derived analytically.

This paper extends the previous studies in several directions and solves the model by using numerical methods. It derives the full dynamics of a balance-of-payments crisis under capital controls or under a dual exchange rate. The crisis is caused by an increase in the fiscal deficit or by a reduction in the devaluation rate. In both cases, households are faced with excess money holdings that spill over to the trade balance. This paper analyzes the dynamics of this spill-over effect and shows how it depends on the preference structure. In particular, it shows that the current account can be improving before the anticipated devaluation, while money holdings can be growing at an increasing rate.

Other aspects of the economy are also analyzed. The path of the dual exchange rate is derived, as well as the returns on short-term nominal and medium-term indexed domestic bonds. It is shown for example that the domestic short-term real interest rate can be higher than the foreign rate even though controls are binding on capital outflows. Capital controls or a dual exchange rate system are shown to always postpone the collapse of the fixed exchange rate.

Finally, the welfare effect of restricted capital mobility is examined. It is shown that capital controls do not necessarilly decrease welfare in a balance-of-payments crisis. Moreover, if the government has a higher discount rate than the individuals, it may find optimal to impose capital controls or a dual exchange rate even when it decreases the individuals' welfare.

The model used is based on intertemporally optimizing, infinitely-lived individuals and is similar to Obstfeld (1986a). Money is included in the utility function. As shown by Feenstra (1986), this representation is quite general and does in particular include the case of a cash-in-advance constraint. Calvo (1989) uses a similar model to study an *exogenous,* but *anticipated* devaluation (2).

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the analytical solution and briefly describes the numerical solution procedure. Section 4 presents the dynamics of the economy before the devaluation, looking in particular at money holdings, the current account deficit, the dual exchange rate, asset returns and the timing of the devaluation. Section 5 looks at welfare and Section 6 extends the analysis to other policies, including a flexible exchange rate system. Section 7 offers concluding remarks.

2. THE MODEL

The framework is a small open economy similar to the one presented by Auernheimer (1987) or Obstfeld (1986a), but including a richer set of assets. This section presents

(2) Employing a similar framework, Obstfeld (1986a) examines an *exogenous, unanticipated* devaluation; Bacchetta (1990) analyzes *temporary* controls; Penati and Pennacchi (1989) and Claessens (1991) introduce uncertainty in domestic credit.

the model in the case of a fixed exchange rate, where the policy is an increase in government transfers leading to a fiscal deficit. Section 6 describes other policies, as well as the case of a flexible exchange rate system.

2.1. The private sector

There is an immortal representative individual optimizing his consumption and portfolio allocation over time. His portfolio wealth is composed of four assets: real money holdings m (3), short therm domestic bonds b, medium term domestic indexed bonds lb (4), and foreign bonds f. The short term domestic bond, with a constant unit price, gives an instantaneous nominal interest rate i. The indexed bond lb(t) is a zero coupon bond: it is issued at time t, is of maturity M and gives a real return h at maturity (i.e. yields h (t + M)). It has also constant unit price. At any time t, the individual holds a stock slb of indexed bonds of various maturities, issued from t – M to t:

$$slb(t) = \int_{t-M}^{t} lb(s) \cdot ds$$

Both short term and medium term domestic bonds are assumed in fixed quantity for the aggregate economy. Finally, the foreign bond is in quantity f* expressed in foreign currency and has a real price q. In domestic currency the quantity is then $f = q \cdot f^*$. It yields an instantaneous interest rate r expressed in foreign currency and repatriated at the fixed exchange rate. With capital controls, the quantity of foreign assets is rationed by the government and supposed fixed: capital flows can occur, but net flows are in fixed quantity. As described by Obstfeld (1986a), this means that q represents a floating *dual exchange rate* applying to capital account transactions (current account transactions being conducted at the fixed exchange rate) (5). Portfolio wealth a is then: $a = m + b + f + slb$.

On the real side, there is a single perishable good traded internationally and produced domestically in constant quantity y. Purchasing power parity is assumed to hold and the foreign price level is assumed to be equal to one. The domestic price level P is therefore equal to the exchange rate and the inflation rate π is equal to the rate of depreciation of the domestic currency.

Domestic money is used for transactions purposes and is assumed to give some indirect utility. The consumer then maximizes his lifetime utility which is assumed to be of the form (6):

$$V = \int_{0}^{\infty} e^{-\delta t} \cdot U(c, m) \cdot dt \qquad [1]$$

where δ is his rate of time preference and U is assumed to be twice differentiable, strictly concave, and increasing in both its arguments and satisfy the Inada conditions. It will be assumed throughout that $\delta = r$.

(3) Lower case letters represent *real* variables at time t, unless otherwise specified.
(4) Medium-term indexed bonds are included to represent real assets with a maturity including the devaluation.
(5) Under full capital mobility, the domestic price of foreign assets is equal to the foreign price, i.e. $q = 1$ and $\dot{q} = 0$.
(6) See Feenstra (1986) for a justification. This formulation gives a cash-in-advance constraint when there is no substitution between money and consumption.

The individual's budget constraint is:

$$\dot{a} = (i - \pi) \cdot b + ((r + \dot{q})/q) \cdot f + h \cdot lb\,(t - M) + y + \tau - c - \pi \cdot m \qquad [2]$$

i.e. portfolio wealth can be modified by the difference between total income and consumption. Total income is composed of the return on the various assets, of output y, and of government transfers τ. Finally, a standard solvency condition is imposed:

$$\lim_{t \to \infty} a(t)\, e^{-rt} \geq 0 \qquad [3]$$

2.2. The government

The government gives a constant real transfer τ to the individual, consumes a quantity g of the good, and receives the interest on the central bank's foreign reserves. The central bank is assumed to finance a possible deficit. The government budget constraint is then:

$$g + \tau - (r + \pi)k = \dot{d} + \pi d \qquad [4]$$

where d represents domestic credit and k the foreign exchange reserves held at the central bank (7). Money supply is:

$$m = k + d \qquad [5]$$

and its change is defined as:

$$\dot{m} = \dot{k} + \dot{d} \qquad [6]$$

Using [4] to [6], the change in foreign reserves can be written as:

$$\dot{k} = rk + \dot{m} + \pi m - \tau - g \qquad [7]$$

For simplicity, it is assumed that the government pegs the exchange rate (8), thus setting $\pi = 0$, and that it has no real expenditures, i.e. $g = 0$. Before time 0, the government budget is assumed to be balanced so that $\tau_0 = rk_0$, where k_0 is the level of reserves in $t = 0$. From time 0, transfers are increased by ε, leading to a budget deficit. In $t = 0$:

$$\tau = rk_0 + \varepsilon \qquad [8]$$

From [4], this means that the deficit in $t = 0$ is equal to ε. The previous studies showed that this situation is not sustainable, as foreign reserves are used to maintain the exchange rate value. It is assumed here that the government abandons the fixed rate when the level of reserves reaches zero (9). The system will then switch to a floating exchange regime at time T such that:

$$k(T) = 0. \qquad [9]$$

(7) Note that the central bank's foreign reserves are the sole government assets.
(8) This means that it cannot control the money stock m as reserves k are used to sustain the fixed rate.
(9) This is the usual assumption made in the balance-of-payments literature, starting from Krugman (1979).

3. SOLVING THE MODEL

3.1. The Analytical Solution

With full capital mobility, the money-financed deficit leads to capital outflows as the individuals buy foreign assets with their excess money holdings. This leads to a decrease in foreign reserves and the fixed exchange rate must be abandoned when the reserves are depleted. The timing of the collapse is determined by equations [7] and [9]. As it is well known from the balance-of-payments crisis literature, there is a speculative attack just before T.

With full capital mobility the individual simply maximizes [1] subject to [2] and [3]. Both money holdings and consumption are constant between 0 and T and after T. Because of the speculative attack in T, money holdings jump down at T. Consumption can jump up or down depending on preferences. If the utility function is separable in money and consumption ($U_{cm} = 0$), the latter is not affected by the crisis and remains constant (10).

With no capital mobility, the individual cannot buy foreign assets and tries instead to buy foreign goods in order to reduce his increased money holdings. This leads to a current account deficit, thereby depleting official foreign reserves. In T, no speculative attack can occur. Again, the only way to reduce money holdings is to spend the excess money on goods. However, as foreign reserves are depleted, net imports are no longer possible and only domestic production y can be consumed. The equilibrium in the goods market is then achieved by an increase in the price and in the exchange rate level at T. There is therefore an *endogenous devaluation* in T (11). After T, the exchange rate floats so that both the current account and the capital account are balanced. Thus, capital account restrictions are no longer binding after T.

The behavior under capital controls is therefore more complex than under capital mobility. In particular, there is a devaluation at T, which means a jump in wealth a. Although the standard procedures cannot be used to derive the optimal solution, a similar approach can be taken. It is possible to form a Lagrangian as in the full capital mobility case [e.g. see Obstfeld (1986a)] (12). The idea is to describe the budget constraints before and after the jump at T and to link them by introducing explicitly the magnitude of the devaluation at time T. Then the problem is solved in a compact way to derive the optimal behavior in the two periods (13).

The solution is fully described in the Appendix and only the main elements are presented below. It is first useful to introduce an artificial separation of time at T : T^- is the moment just before the devaluation and T^+ is right after the devaluation. If the magnitude of the devaluation is κ, the real value of nominal domestic assets declines proportionately to κ at T. For real money holdings we have:

$$m(T^+) = (1 - \kappa)\, m(T^-) \qquad [10]$$

(10) More details about the capital mobility case can be found in Obstfeld (1985, 1986b) and in Bacchetta (1988).
(11) Penati and Pennacchi (1989) show that in a similar framework the presence of uncertainty under capital mobility can also lead to a devaluation.
(12) See however Bacchetta (1987, 1988) for a solution using optimal control.
(13) It should be noticed that Calvo (1988) uses a similar approach and justifies it carefully.

As indexed bonds do not lose value when a devaluation accurs, the evolution of financial wealth at T is:

$$a(T^+) = (1-\kappa)\, a(T^-) + \kappa \cdot slb(T) \qquad [11]$$

It is then possible to compute the entire budget contraint from 0 to ∞:

$$0 = e^{-rT} \cdot \left\{(1 - \kappa) \cdot \left[a(0) \cdot e^{\int_0^{T^-} i(s) \cdot ds} + \int_0^{T^-} e^{\int_t^{T^-} i(s) \cdot ds} X1(t) \cdot dt\right] + \kappa \cdot slb(T)\right\} +$$
$$+ \int_{T^+}^{\infty} e^{-rt} X2(t) \cdot dt \equiv W \qquad [12]$$

The individual maximizes [1] subject to [12]. To solve this problem, it is possible to use the Lagrangian L:

$$L = V + \lambda \cdot W \qquad [13]$$

where λ is the Lagrange multiplier (and is a constant). The Appendix derives the first orden conditions in details. Several useful results can be derived from these conditions and are described below; first for c and m after T:

$$U_m = (r + \pi) \cdot U_c \qquad t > T \qquad [14]$$
$$U_c = \lambda \qquad t > T \qquad [15]$$

where [14] is derived from [A8] and [A11] and [15] is derived from [A8]. [14] and [15] mean that c and m are constant after T. Not surprisingly the behavior after T is similar to the case with full capital mobility, as the controls are not binding. At time T, from [A9] and [15] we have:

$$U_c(T^-) = (1 - \kappa) \cdot U_c(T^+) \qquad [16]$$

Equation [16] links consumption right before the devaluation, $c(T^-)$, with consumption right after, $c(T^+)$. It means that a dollar consumed in T^- gives the same marginal utility as if this dollar is held and consumed in T^+.

The most interesting behavior is before the devaluation. The behavior of c and m before T is described by the following system:

$$\dot{c} = (r \cdot U_c - U_m - \dot{m} \cdot U_{mc})/U_{cc} \qquad [17]$$
$$\dot{m} = y + \tau - c \qquad [18]$$
$$m(0) = m_0 \qquad [19]$$
$$U_c(T^-) \cdot m(T^-) = U_c(T^+) \cdot m(T^+) \qquad [20]$$

Equation [17] is derived from [A7] and [A10]. The behavior of money holdings in [18] is derived from [2], recalling that other assets are in fixed supply, i.e. $\dot{b} = \dot{lb} = \dot{f} = 0$. The last equality is the assumption of restricted capital mobility and means that there is no net capital outflow or inflow. This assumption is rather extreme and must be seen as a benchmark case (like the case of full capital mobility). Other types of restrictions certainly deserve attention, but are technically more complex. In [18], it is also assumed that all interest bearing assets are in zero net supply (14). Finally, equation [19] follows

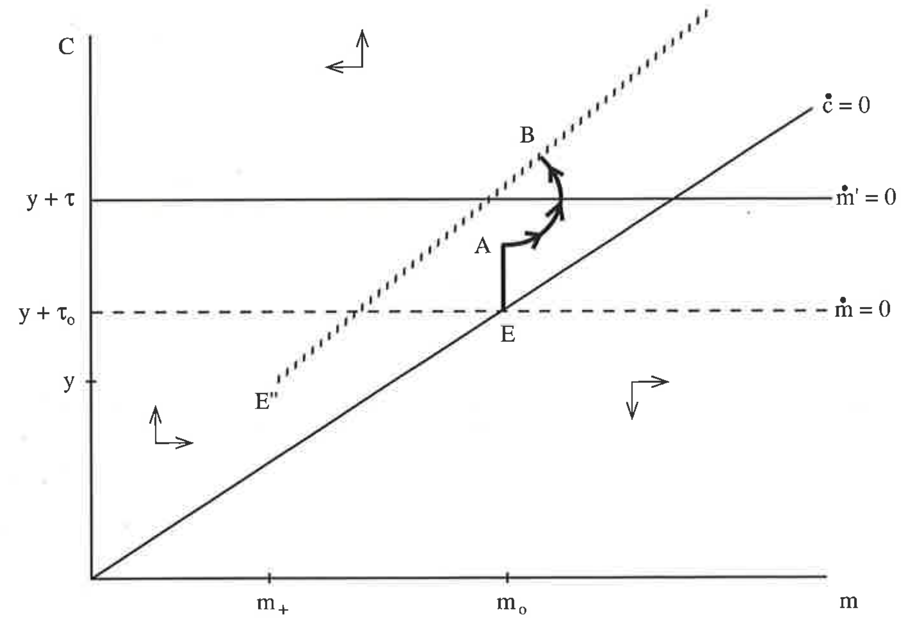
(14) This last assumption does not affect the results is any significant way.

from the assumption of no capital mobility and equation [20] is derived from [10] and [16].

Equations [17], [18], and [19] are similar to systems with money in the utility function [e.g. Calvo (1981)], but equation [20] is a non-linear terminal constraint that modifies the problem significantly.

The system [17] to [20], representing the behavior of c and m before T, can be depicted on a phase diagram as in Figure 1. At T = 0, the system is at point E where the $\dot{m} = 0$ and the $\dot{c} = 0$ schedules initially cross. When the monetary transfers are increased, the $\dot{m} = 0$ schedule shifts up by ε, to $\dot{m}' = 0$. The system is then driven by the new laws of motion, but does not converge to the new steady state: the situation is not sustainable and a devaluation occurs at time T. At T, c and m jump down in a proportion given by [20]. This proportion is represented in Figure 1 by the broken line starting from E".

Figure 1
Phase diagram for c and m under no capital mobility



A possible behavior of c and m is shown in Figure 1. At time 0, c jumps from E to A. m and c then move to B until T and jump down to E" at time T. This means that after the initial jump, c continues to increase, i.e. there is an increasing current account deficit. Concerning money holdings, they first increase as more transactions are conducted. As the time of the devaluation comes closer, however, the individual reduces his money holdings as he is going to lose on them. This behavior was conjectured by Bacchetta (1987) and is similar to the one conjectured by Auernheimer (1987). Numerical solutions will show that this behavior is possible, but that strikingly different types of behavior can occur.

Once the behavior of c and m is known, other variables can be derived recursively. First, the behavior of foreign exchange reserves can found by substituting equation [18] into [7]:

$$\dot{k} = rk + y - c \qquad [21]$$

As there are no net private capital flows, the change in reserves is equal to the current account.

The return on the various assets can be determined from the first order conditions. First, the behavior of the short-term interest rate can be found from [A7] and [A10]:

$$i = U_m / U_c \qquad [22]$$

For the dual exchange rate q, at any time $t < T$ its value is (from [A14]):

$$q(t) = \frac{1}{1-\kappa} \cdot e^{-\int_t^{T^-} i(u) \cdot du} + r \cdot \int_t^{T^-} e^{-\int_t^s i(u) \cdot du} ds \qquad [23]$$

As $q = 1$ from T, its value before T is equal to the discounted loss on domestic assets plus the discounted value of interest payments from t to T.

Concerning the return on indexed bonds h, from [A19] its value before t when $t + M > T$, (i.e. when the maturity includes the devaluation) can be written as:

$$h(t+M) = \int_t^{t+M} e^{\int_s^{t+M} \rho(u) \cdot du} \rho(s) \cdot ds - e^{r(t+M-T)} \cdot \kappa \cdot \left(1 + \int_t^{T^-} e^{\int_s^{T^-} i(u) \cdot du} i(s) \cdot ds\right) \qquad [24]$$

where ρ is equal to the real return on short term domestic bonds, i.e. $\rho = i$ before T and $\rho = r$ after T. The return on long term domestic indexed bonds is equal to the stream of short term interest between t and t + M, minus the loss in the value of the domestic bond and in the accumulated interests between t and T^-.

As mentioned above, the full solution of the program cannot be derived analytically as T and κ are determined endogenously (although they are taken as given by the individual). The next sub-section briefly describes how to solve the problem numerically.

3.2. The Numerical Solution

3.2.1. *Parameterizing the model and specifying the utility function*

To compute the simulations, it is necesary to specify a functional form for the utility function and to give a value for a few parameters. It is assumed that both the elasticity of intertemporal substitution and the elasticity of substitution between money and consumption are constant. The instantaneous utility function is thus a generalized CES function:

$$U(c, m) = \frac{v(c, m)^{1-\alpha}}{1-\alpha} \qquad [25a]$$

where:

$$v(c, m) = \left[\beta \cdot c^{1-\rho} + (1-\beta) \cdot m^{1-\rho}\right]^{\frac{1}{1-\rho}} \qquad [25b]$$

The coefficient α is the coefficient of relative risk aversion and $1/\alpha$ is the intertemporal elasticity of substitution. $1/\rho$ is the elasticity of substitution between c and m. It also represents the interest elasticity of money demand.

This utility function is quite general. Some particular cases are:

(i) When $\rho = 2$, we have: $U(c, m) = \dfrac{(c^{\beta} \cdot m^{1-\beta})^{1-\alpha}}{1-\alpha}$

(ii) When $\alpha = \rho$, $U_{cm} = 0$, i.e. the function is separable between c and m. In particular, when $\alpha = \rho = 1$ we have the standard log utility function: $U(c, m) = \beta \cdot \ln c + (1-\beta) \cdot \ln m$.

(iii) When $\rho \to \infty$, we have no substitution between money and consumption and we have: $v(c, m) = \min[c, m]$

This is the standard cash-in-advance technology.

The simulations are performed for various types of utility functions: α is varied between 1 and 25 and ρ between 1 and 125 (15). The other parameters are (unless otherwise specified):

$$y = 100, \qquad k_0 = 60, \qquad r = 0.01, \qquad \varepsilon = 5.$$

A time period is thought to be a month. The foreign exchange reserves represent 5% of annual GDP and the budget deficit is increased by 5% of GDP. The coefficient β is determined such that $m/c = 3$. The other initial variables are determined from the various behavioral and budget constraints equations.

3.2.2. *A simple shooting algorithm*

Although the problem is too complex to be solved analytically, the main variables behave smoothly and a stable solution is easily reached by simulation methods. The cen-

(15) Empirical studies have found values for α between 1 and 7. The interest elasticity of money demand is typically around 0.1, thus giving a value of $\rho = 10$.

tral problem is to derive the behavior of c and m as described by the system [17] to [20] and depicted in figure 1. This actually represents a *two-point boundary value problem* and can be solved by simulating the paths of c and m such that they satisfy the necessary conditions (16). This approach is commonly used in rational expectations models and several algorithms have been developed, in particular multiple shooting [see Lipton et al. (1982)] and the Fair-Taylor method [Fair and Taylor (1983)]. As our particular case behaves smothly and involves few variables, a simple shooting method is sufficient [see Press et al. (1986)]. It is also computationally faster. The idea is to give an initial value c(0) for the jumping variable c (in $T = 0$), to compute the paths for c and m and to check whether the terminal condition [20] is satisfied. This process is iterated by using a Newton-Raphson algorithm until the right c(0) is found. This is done however for a given T, but T should be found such that $k(T) = 0$ is satisfied. The process is repeated for various T's and the solution is reached by a grid search method. Once c, m and T are found, other interesting variables such as interest rates and the dual exchange rate can be solved recursively.

4. BALANCE-OF-PAYMENTS DYNAMICS UNDER CAPITAL CONTROLS OR A DUAL EXCHANGE RATE

4.1. Consumption, Real Money Holdings, and the Current Account Deficit

Section 3 showed that an increase in the fiscal deficit leads to potential excess money holdings. Under full capital mobility, the individual can buy foreign assets, and therefore holds his desired level of money holdings. Under capital controls, however, the only way to reduce money holdings is by consuming more and in particular more imported goods. The spill-over from excess money holdings to the trade balance should depend on preferences and in particular on two factors:

i) The degree of substitutability between money and consumption.

ii) The degree of intertemporal substitutability of consumption.

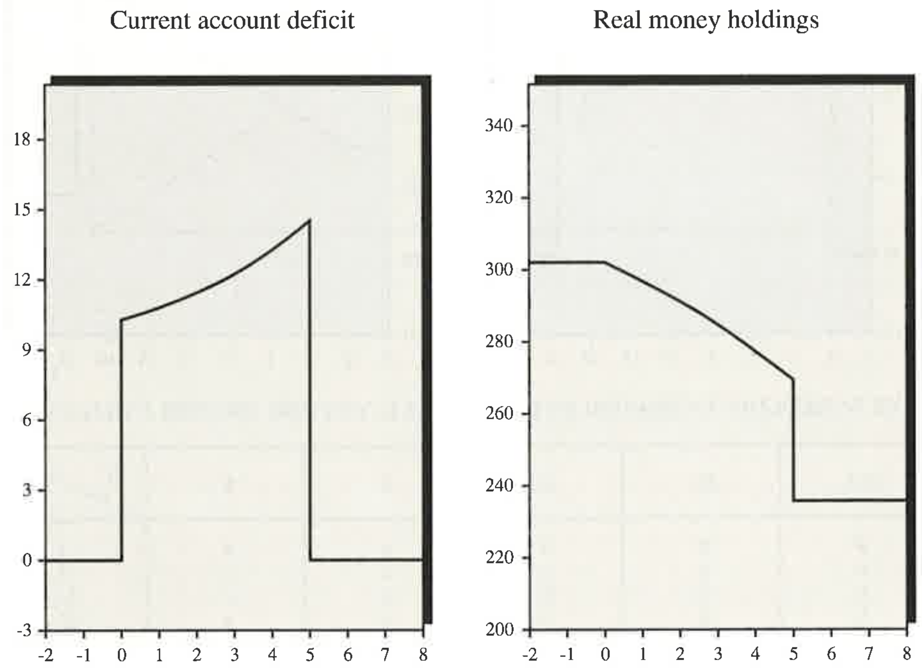
If money and consumption are close substitutes, the spill-over effect is large, i.e. there is a large increase in consumption and in the trade deficit. If consumption today is a close substitute of consumption tomorrow, the path of consumption need not be as smooth and the increase in consumption and in the trade deficit can be larger. In the utility function [25], $1/\rho$ is the elasticity of substitution between consumption and money and $1/\alpha$ is the intertemporal elasticity of substitution. Thus, when either ρ or α increases, the spill-over effect is smaller.

A different behavior of consumption leads to a completely different behavior of money holdings. Section 3 presented the case where money was first increasing and then decreasing. When the elasticities of substitution are large, we can even have money holdings decreasing all allong. Figure 2 shows the behavior of the current account deficit and of money holdings when $\alpha = 1$ and $\rho = 5$.

(16) It would also be possible to use a linear approximation of the model.

When α and ρ are larger, however, the current account deficit is smaller and money holdings can be increasing all along. Figure 3 shows such a case, when $\alpha = 10$ and $\rho = 25$. A lower elasticity of substitution between m and c in particular means that the initial jump in the current account deficit is smaller (17). A surprising feature is that the current account is improving while money holdings are increasing despite the anticipated loss from the devaluation. With a low elasticity of intertemporal substitution, the individual wants a smoother consumption path. As he knows that there will be a downward jump in consumption he will decrease it before the devaluation (18).

Figure 2
$\alpha = 1, \quad \rho = 5$

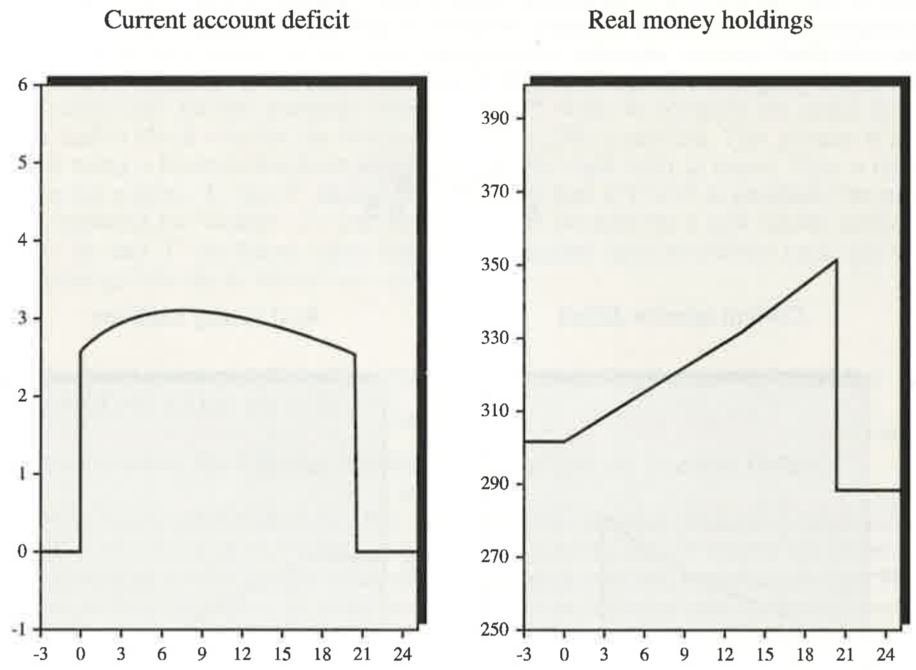


$\alpha = 1$

$\rho = 5$

(17) At the limit, when $\rho \to \infty$, there would be no initial jump. [see Park and Sachs (1987)].
(18) At the limit, when $\alpha \to \infty$, there would be no jump.

Figure 3
$\alpha = 10, \quad \rho = 25$



$\alpha = 10$

$\rho = 25$

The latter behavior can be understood on the phase diagram in Figure 4 similar to Figure 1 (for the case of $U_{cm} = 0$). c jumps from E to C; then the system moves from C to D and jumps to E" in T. With higher α and ρ, the terminal constraint [20] becomes «flatter» which explains the difference in behavior.

Table 1 shows the different types of behavior for a larger range of coefficient values. It shows that the actual dynamics are strongly influenced by preferences (19).

(19) Obsiously, the actual behavior is also influenced by the other parameters of the economy, in particular by the size of the fiscal deficit and the initial level of foreign exchange reserves.

Figure 4
Phase diagram for c and m under no capital mobility-Alternative behavior

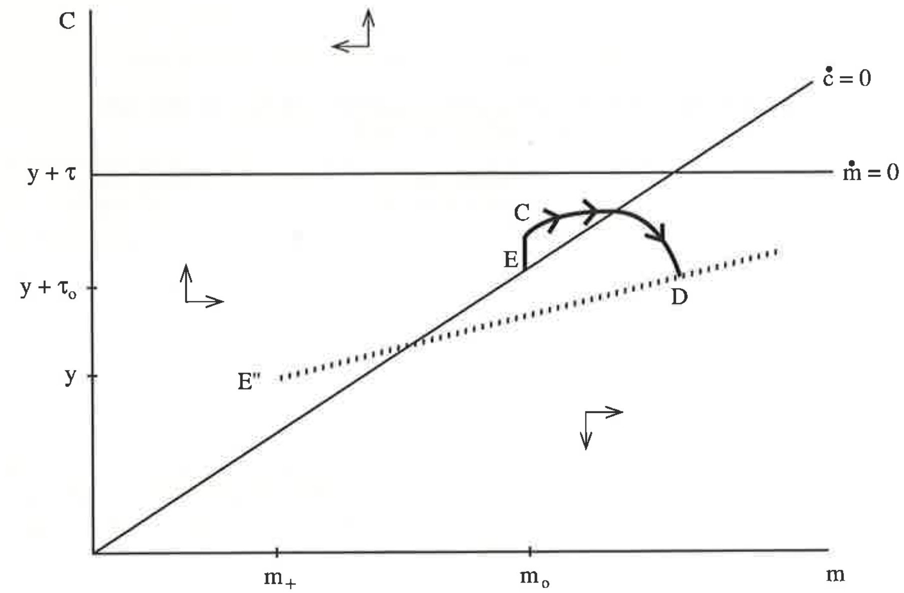


Table 1

DYNAMICS BEFORE THE DEVALUATION WITH DIFFERENT PREFERENCES

α \ ρ	1	5	10	25	125
1	a	a	b	b	b
5	a	c	d	d	d
10	a	d	d	d	d
25	e	e	e	d	d

with:
a: the current account deficit (CAD) increases; m decreases
b: the CAD increases; m increases and then decreases
c: the CAD increases; m increases
d: the CAD first increases, then decreases; m increases
e: the CAD decreases (after an initial jump); m increases.

4.2. The Timing of the Collapse

The timing of the collapse also depends on preferences as it is determined by the loss of reserves caused by the current account deficit. Table 2 shows how the timing increases as ρ and α increase: as the current account deficit is smaller when α and ρ increase, the timing is longer. It should be noticed that the differences are large: it varies between .2 in case 1 and 24.9 in case 4.

Table 2

THE TIMING OF THE COLLAPSE UNDER CAPITAL MOBILITY AND CAPITAL CONTROLS

Various cases	Timing under capital controls	Timing under capital mobility
1. $\alpha = \rho = 1$	.23	0
2. $\alpha = \rho = 5$	12.21	0
3. $\alpha = \rho = 10$	19.87	5.43
4. $\alpha = \rho = 25$	24.94	9.03
5. $\alpha = \rho = 5,\ k_0 = 600$	82.30	69.81

The second column of Table 2 shows the timing under capital mobility. It is always shorter than under capital controls (20). In particular, the timing may be zero with capital mobility as a speculative attack occurs at the moment the fiscal deficit begins. Such a stock adjustment is not possible under capital controls.

Case 5 represents a case where initial foreign reserves are large (50% of annual GDP). It shows that capital controls are more useful in postponing the collapse when the level of reserves is smaller.

4.3. Asset Returns and the Dual Exchange Rate

As asset returns depend on c and m, they must also be derived by numerical methods. The behavior of these returns shows surprising results. Equations [22] to [24] describe the paths of the short term interest rate, the return on medium-term indexed bonds, and the dual exchange rate. They are shown on Figure 5 in the case where $\alpha = \rho = 5$ and their behavior is discussed more generally below.

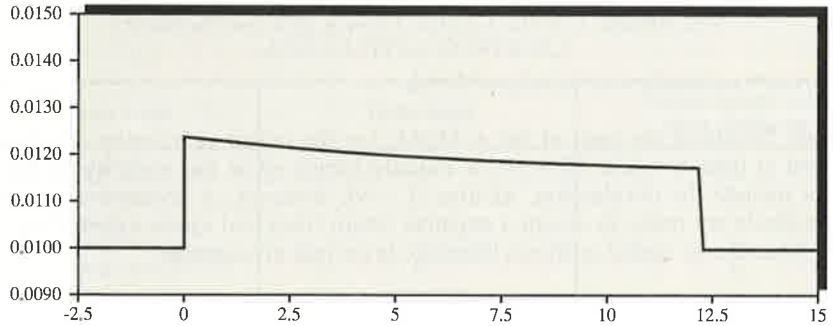
i) *The short-term real interest rate*

Figure 5a shows the path of $i - \pi$. The striking feature is that *the domestic interest rate is higher than the foreign rate when controls are binding on capital outflows* (21).

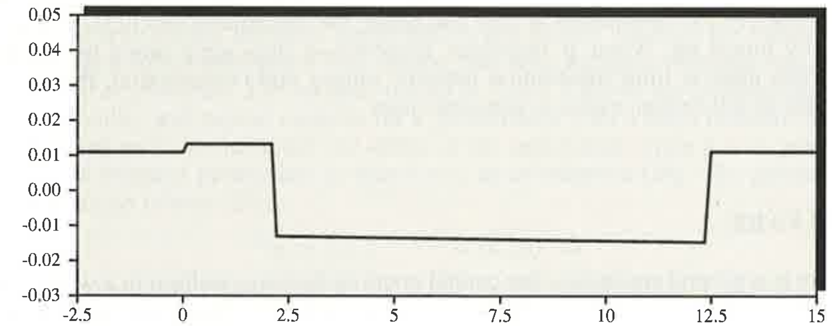
(20) This was shown theoretically by Park and Sachs (1987) in the particular case where $\alpha = 1$ and $\rho \to \infty$.
(21) A domestic interest rate higher than abroad before a devaluation is usually associated with capital mobility and

Figure 5

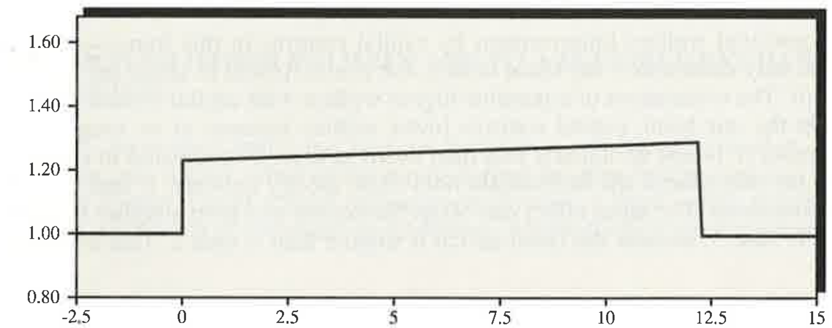
a. Real short term interest rate on domestic nominal bonds



b. Return on medium term domestic indexed bonds



c. Dual exchange rate



This happens because the individual would like to sell his domestic bonds to consume more. On Figure 5a, the short-term rate first jumps up and then decreases. More generally, an upward jump can always be observed, but the behavior before the devaluation depends on preferences. When α and ρ are large, the domestic rate may become *lower* than the foreign rate just before the devaluation, as consumption is decreasing (22).

ii) *The return on medium-term indexed bonds*

Figure 5b shows the path of h(t + M)/M, i.e. the return at maturity of a bond lb purchased at time t, when M = 10. h initially jumps up as the maturity of the bond does not include the devaluation. At time T – M, however, h becomes negative; i.e. the individuals are ready to accept a negative return from real assets to avoid the devaluation. Controls on capital outflows therefore favor real investments.

iii) *The dual exchange rate*

Figure 5c shows the behavior of q. Not surprisingly, the price of foreign assets increases before the devaluation as they can hedge the consequent loss (23). In Figure 5c, q initially jumps up. When ρ increases, however, q may jump down before increasing. When there is little substitution between money and consumption, the individual would like to sell foreign assets to consume more.

5. WELFARE

There is a general consensus that capital controls decrease welfare in a world without distortions, as they restrict intertemporal trade for a country. This result could be expected in a typically neo-classical framework. It turns out, however, that this is not necessarily the case. The firs column of Table 3 shows the comparison of welfare between capital mobility and capital controls. Welfare is improved by capital controls in case 5 with a smaller fiscal deficit (2% of GDP) than in the benchmark case.

The potential welfare improvement by capital controls in this framework is surprising. The only distortion is the fiscal deficit, but it is sufficient to affect the general welfare result. The explanation of a possible higher welfare with capital controls is the following. On the one hand, capital controls lower welfare because of an irregular path of consumption (c jumps up initially and then down at time T) compared to capital mobility. On the other hand, the individuals hold more money between 0 and T, reducing transactions costs. The latter effect can offset the former and give a higher welfare. This happens in case 5, because the fiscal deficit is smaller than in case 2. This means that the

(22) It can be shown theoretically that the domestic rate must always be higher than the foreign rate on average.
(23) The price of foreign assets thus behaves similarly to the price of domestic real assets.

devaluation comes at a later date and the time period during which the transactions costs are reduced is longer.

Table 3

COMPARING WELFARE UNDER CAPITAL MOBILITY AND CAPITAL CONTROLS

Various cases	Individual	Government with short horizon
1. $\alpha = \rho = 1$	a	a
2. $\alpha = \rho = 5$	a	b
3. $\alpha = \rho = 10$	a	b
4. $\alpha = \rho = 25$	a	b
5. $\alpha = \rho = 5,\ \varepsilon = 2$	b	b

with:
a: capital mobility is preferred to capital controls
b: capital controls are preferred to capital mobility.

Compared with capital mobility, consumption and money holdings are higher before the collapse and lower thereafter. This means that with a higher discount rate, capital controls could be preferred. The second column of Table 3 shows the comparison between capital mobility and capital controls for a government with a short horizon. Suppose the government wants to maximize the utility of the individuals while it is in power, but that there is a constant probability p that it will be overthrown (24). The government's objective function is then (25):

$$V_G = \int_0^\infty e^{-(\delta+p)t} \cdot U(c, m) \cdot dt \qquad [26]$$

The second column of Table 3 shows the results when the probability of being overthrown is 0.1, i.e. the expected lifetime of a government is 10 periods. It shows that in three cases (2 to 4), the government would impose capital controls or a dual exchange rate even though they give a lower welfare to the individual.

6. EXTENSION TO OTHER POLICIES AND TO A FLEXIBLE EXCHANGE RATE SYSTEM

The policy considered so far has been an increase in government transfers under a fixed exchange rate regime. Most of the results extend to other policies and to a flexible exchange rate system. Under a fixed exchange rate system, a fiscal deficit could also be caused by an increase in government expenditures. Alternatively a similar situation can

(24) A full incorporation of the government's behavior is of considerable interest, but is beyond the scope of this analysis.
(25) See Blanchard (1985) for an application of finite horizons to the individual.

arise when the central bank reduces the devaluation rate in an active crawling peg system. Both cases give results similar to those presented in Sections 4 and 5.

The framework could also be extended to a flexible exchange rate system where the central bank sets the rate of money growth [see Auernheimer (1987)]. If the money growth is too small to finance a possible budget deficit, the government will have to use its foreign assets or to borrow abroad. This is clearly not sustainable and there will be a point where the central bank must abandon its monetary policy. The unsustainable situation could result from a larger fiscal deficit caused by an increase in monetary transfers or in government expenditures, or by a decrease in the rate of money growth.

Although there are some differences in the economic mechanisms, most of the results under a fixed exchange rate can be extended to a flexible exchange rate system. When the monetary policy is not sustainable, the individual knows that there will be a higher money growth and a higher inflation rate in the future. Thus he would like to reduce his money holdings by buying foreign assets. Under capital controls or a dual exchange rate this is not possible and there is a spill-over to the trade balance. The actual behavior of real money balances before the devaluation again depends on preferences, in the way described in Section 4. The behavior of asset returns is also similar to the fixed exchange rate case; in particular, the domestic short-term interest rate can be higher than the foreign rate when a higher monetary growth is expected.

7. CONCLUSION

This paper has shown that a dual exchange rate, or capital controls dramatically affect the dynamics of a balance-of-payments crisis. In particular, the role of preferences becomes more important than under capital mobility and this role has been analyzed in details.

The analysis is based on a bare-bone model that abstracts from many important aspects. Some assumptions of the model can be easily relaxed. For example, alterable stores of value, such as imported durable goods could be added to money. The behavior of money holdings is then likely to be less dramatic. The assumption of purchasing power parity could also be relaxed and nontradable goods could be introduced. This would lead to an appreciation of the real exchange rate.

Other aspects, however, may be more difficult to introduce and deserve further investigation. Important issues neglected in this analysis are: the role of foreign residents, the role of financial intermediation, or the existence of excess capacity. A particularly extreme assumption made is perfect foresight. The role of uncertainty, especially about government policies, should be taken into account.

Finally, this paper has shown the usefulness of numerical methods in a small model. In many theoretical studies based on macroeconomic foundations, strong assumptions about preferences are usually made to derive an analytical solution. This paper has shown, however, that preferences can significantly affect the results. In a complex analytical framework, the use of numerical methods may be more adequate than drastically simplifying assumptions: relevance should be preferred to elegance.

APPENDIX

This appendix solves explicitly the optimization problem described in Section 3, with a fixed exchange rate and an increase in government transfers under no capital mobility.

Before T, the individual budget constraint is (from [2]):

$$\dot{a} = i \cdot a + X1 \qquad [A1]$$

with $X1 = ((r + \dot{q})/q - i) \cdot f - i \cdot slb + h \cdot lb\,(t - M) - i \cdot m + y + \tau - c$.

After T, the rate of inflation π is positive and individuals feel no longer constrained to buy foreign assets. This means that we are in a case similar to full capital mobility. Hence $q = 1$ and $\dot{q} = 0$, and $i = r + \pi$. Using these results, the budget constraint after T is:

$$\dot{a} = r \cdot a + X2 \qquad [A2]$$

with $X2 = r \cdot slb - (r + \pi)m + h \cdot lb\,(t - M) + y + \tau - c$.

With no capital mobility, a devaluation of magnitude κ occurs at T and nominal domestic assets decline proportionately to κ at T. In addition to [10] we have, first for short-term domestic bonds:

$$b(T^+) = (1 - \kappa)\, b(T^-) \qquad [A3]$$

Arbitrage between b and f in T means that q jumps by the same amount:

$$q(T^+) = (1 - \kappa)\, q(T^-) \qquad [A4]$$

therefore:

$$f(T^+) = (1 - \kappa)\, f(T^-) \qquad [A5]$$

By using the definition of financial wealth a and the fact that index bonds do not lose value when a devaluation occurs (i.e. $slb(T^+) = slb(T^-)$) we get equation [11]. Equation [12] can be found by integrating [A1] from 0 to T^- and [A2] from T^+ to ∞, linking the two by [11], and using [3].

By maximizing [1] subject to [12], we can form the Lagrangian L in [13]. Using the Lagrangian method enables a compact and straightforward derivation of the various necessary conditions. L can be written in full:

$$\begin{aligned} L = {} & \int_0^{T^-} e^{-rt} \cdot [u(c) + v(m)]dt + \int_{T^+}^{\infty} e^{-rt} \cdot [u(c) + v(m)]dt \\ & + \lambda \cdot \Big[e^{-rT} \cdot \Big\{ (1 - \kappa) \cdot \Big[a(0) \cdot e^{\int_0^{T^-} i(s) \cdot ds} + \int_0^{T^-} e^{\int_t^{T^-} i(s) \cdot ds} \cdot \{((r + \dot{q})/q - i) \cdot f \\ & - i \cdot \int_{t-M}^{t} lb(s) \cdot ds + h \cdot lb\,(t - M) - i \cdot m + y + \tau - c\} \cdot dt \Big] + \kappa \cdot \int_{t-M}^{T-} lb(s) \cdot ds \Big\} \\ & + \int_{T^+}^{\infty} e^{-rt} \cdot \{-r \int_{t-M}^{t} lb(s) \cdot ds - (r + \pi)\, m + h \cdot lb\,(t - M) + y + \tau - c\} \cdot dt \Big] \end{aligned} \qquad [A6]$$

The first order conditions can then be derived, first concerning c:

$$\frac{\partial L}{\partial c\,(t)} = e^{-rt} \cdot u' - \lambda \cdot e^{-rT} \cdot (1-\kappa) \cdot e^{\int_t^{T^-} i(s)\cdot ds} = 0 \qquad 0 \le t \le T^- \qquad [A7]$$

$$\frac{\partial L}{\partial c\,(t)} = e^{-rt} \cdot u' - \lambda \cdot e^{-rT} = 0 \qquad t \ge T^+ \qquad [A8]$$

In particular at T^- we have:

$$\frac{\partial L}{\partial c-} = e^{-rT} \cdot u' - \lambda \cdot e^{-rT} \cdot (1-\kappa) = 0 \qquad [A9]$$

For m we have:

$$\frac{\partial L}{\partial m\,(t)} = e^{-rt} \cdot v' - \lambda \cdot e^{-rT} \cdot (1-\kappa) \cdot i \cdot e^{\int_t^{T^-} i(s)\cdot ds} = 0 \qquad 0 < t \le T^- \qquad [A10]$$

$$\frac{\partial L}{\partial m\,(t)} = e^{-rt} \cdot v' - \lambda \cdot (r+\pi) \cdot e^{-rT} = 0 \qquad t \ge T^+ \qquad [A11]$$

Concerning foreign assets f, before T we have:

$$\frac{\partial L}{\partial f\,(t)} = \lambda \cdot e^{-rT} \cdot (1-\kappa) \cdot \left((r+\dot{q})/q - i\right) \cdot e^{\int_t^{T^-} i(s)\cdot ds} = 0 \qquad 0 \le t \le T^- \qquad [A12]$$

This implies:

$$(r+\dot{q})/q = i \qquad [A13]$$

Integrating from t to T^- and using [A4]:

$$q(t) = \frac{q(T^+)}{1-\kappa} \cdot e^{-\int_t^{T^-} i(u)\cdot du} + r \cdot \int_0^{T^-} e^{-\int_t^s i(u)\cdot du}\, ds \qquad 0 \le t \le T^- \qquad [A14]$$

With respect to long term bonds, three intervals give a different behavior: before T, but with a maturity not including the devaluation ($T < T - M$); before T, but including a devaluation ($T - M < t < T$); and after T.

First before T, but not including the devaluation:

$$\frac{\partial L}{\partial lb\,(t)} = \lambda \cdot e^{-rT} \cdot (1-\kappa) \cdot \Big[h\,(t+M) \cdot e^{\int_{t+M}^{T^-} i(s)\cdot ds} \qquad [A15]$$

$$- \int_t^{t+M} i(s) \cdot e^{\int_s^{T^-} i(u)\cdot du}\, ds\Big] = 0 \qquad 0 \le t < T - M$$

This implies:

$$h\,(t+M) = \int_t^{t+M} i(s) \cdot e^{\int_s^{t+M} i(u)\cdot du}\, ds \qquad 0 \le t < T - M \qquad [A16]$$

i.e., h is equal to the present value of the stream of short-term interest rates. After T we have:

$$\frac{\partial L}{\partial lb\,(t)} = \lambda \cdot \Big[h\,(t+M) \cdot e^{-r(t+M)} - r \cdot \int_t^{t+M} e^{-rs}\, ds\Big] = 0 \qquad t > T \qquad [A17]$$

Therefore:

$$h\,(t+M) = e^{rM} - 1 \qquad t > T \qquad [A18]$$

When the maturity includes the devaluation:

$$\frac{\partial L}{\partial lb\,(t)} = \lambda \cdot \Big[e^{-rT}\,(1-\kappa) \cdot \int_t^{T^-} i(s) \cdot e^{\int_s^{T^-} i(u)\cdot du}\, ds + e^{-rT} \cdot \kappa \qquad [A19]$$

$$-r \cdot \int_{T^+}^{t+M} e^{-rs}\, ds + h\,(t+M) \cdot e^{-r(t+M)}\Big] = 0 \qquad T - M < t < T$$

This gives equation [24] in Section 3.

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