Online Appendix

Offshore Dollar Funding Shocks and the Dollar Exchange Rate

Philippe Bacchetta, J. Scott Davis and Eric van Wincoop

January 2025

This Online Appendix has 4 sections. Section A derives the optimal portfolio of European households. Section B derives the spot market equilibrium. Section C discusses the pre-shock equilibrium. Section D adds noise traders to the model.

A Portfolio Foreign Households

In period 1 Foreign households hold dollar and euro money balances, as well as euro bonds, and borrow dollars synthetically, from US households and from the ECB. Their period 2 consumption is

$$\begin{aligned} P_2^* C_{F,2} &= Y_{F,2} + \Pi_{FCB,2} + \frac{1}{S_2} M_{F,1}^{\$} + M_{F,1}^{€} + (1 + i^{€}) B_{F,1}^{€} \\ &\quad - \frac{1 + i_1^{\$,syn}}{S_2} D_{F,1}^{\$,syn} - \frac{1 + i^{\$}}{S_2} D_{F,1}^{\$} - \frac{1 + i^{\$,swap}}{S_2} D_{swap,1}^{\$} \end{aligned} \tag{A.1}$$

Using that period 1 financial wealth is equal to

$$W_{F,1} = B_{F,1}^{€} - \frac{1}{S_1} \left(D_{F,1}^{\$,syn} + D_{F,1}^{\$} + D_{swap,1}^{\$} \right) \tag{A.2}$$

we can also write this as

$$\begin{aligned} P_2^* C_{F,2} &= Y_{F,2} + \Pi_{FCB,2} + \frac{1}{S_2} M_{F,1}^{\$} + M_{F,1}^{€} + (1 + i^{€}) W_{F,1} \\ &\quad - \left(\frac{1 + i_1^{\$,syn}}{S_2} - \frac{1 + i^{€}}{S_1} \right) D_{F,1}^{\$,syn} - \left(\frac{1 + i^{\$}}{S_2} - \frac{1 + i^{€}}{S_1} \right) D_{F,1}^{\$} \\ &\quad - \left(\frac{1 + i^{\$,swap}}{S_2} - \frac{1 + i^{€}}{S_1} \right) D_{swap,1}^{\$} \end{aligned} \tag{A.3}$$

Period 2 income is

$$Y_{F,2} = e^{-\kappa_F \theta s_2} \tag{A.4}$$

Using that $p_2^* = -0.5 s_2$, linearizing (A.3) around $C_{F,2} = \bar{C}_{F,2}$, $s_2 = s_1 = 0$ and zero interest rates, we have

$$\begin{aligned} C_{F,2} &= 1 - \rho s_2 + \Pi_{FCB,2} + (1 - s_2) M_{F,1}^{\$} + M_{F,1}^{€} + (1 + i^{€}) W_{F,1} \\ &\quad - (i_1^{\$,syn} - i^{€} - s_2 + s_1) D_{F,1}^{\$,syn} - (i^{\$} - i^{€} - s_2 + s_1) D_{F,1}^{\$} \\ &\quad - (i^{\$,swap} - i^{€} - s_2 + s_1) D_{swap,1}^{\$} \end{aligned} \tag{A.5}$$

where $\rho = \kappa_F \theta - 0.5\bar{C}_{F,2}$.

Agents maximize the following mean-variance objective:

$$EC_{F,2} - 0.5\gamma var(C_{F,2}) \tag{A.6}$$

This is

$$\begin{aligned} &1 + \Pi_{FCB,2} + M^{\$}_{F,1} + M^{€}_{F,1} + (1 + i^{€})W_{F,1} - (i_1^{\$,syn} - i^{€} + s_1)D^{\$,syn}_{F,1} \\ &-(i^{\$} - i^{€} + s_1)D^{\$}_{F,1} - (i^{\$,swap} - i^{€} + s_1)D^{\$}_{swap,1} \\ &-0.5\gamma\left(-\rho - M^{\$}_{F,1} + D^{\$,syn}_{F,1} + D^{\$}_{F,1} + D^{\$}_{swap,1}\right)^2 var(s_2) \end{aligned} \tag{A.7}$$

The first-order condition with respect to $D^{\$,syn}_{F,1}$ gives

$$N^{\$}_{F,1} = M^{\$}_{F,1} - D^{\$,syn}_{F,1} - D^{\$}_{F,1} - D^{\$}_{swap,1} = -\rho + \frac{i_1^{\$,syn} - i^{€} + s_1}{\gamma var(s_2)} \tag{A.8}$$

B Spot Market Equilibrium

In order to derive the spot market equilibrium, we need to track changes in foreign currency positions of the agents.

B.1 European Households

European households enter period 1 with $M^{\$}_{F,0}$ dollar balances, while at the end of period 1 they have $M^{\$}_{F,1}$ dollar balances. During period 1 these dollar balances change due to dollar invoiced income, dollar invoiced consumption, new synthetic dollar borrowing, dollar borrowing from US households, dollar borrowing from the ECB, interest and principal payments on period 0 dollar borrowing and spot market purchases of dollars in period 1.

With regards to income, European households receive dollar income from exports to the US that are invoiced in dollars. This is equal to

$$Y^{\$}_{F,1} = C_{HF,1} + C^{o}_{HF,1} \tag{B.9}$$

Their spending in dollars on goods imported from the US that are invoiced in dollars is $C_{FH,1}$.

We then have

$$
\begin{aligned}
M_{F,1}^{\$} &= M_{F,0}^{\$} + Y_{F,1}^{\$} - C_{FH,1} + D_{F,1}^{\$,syn} + D_{F,1}^{\$} + D_{swap,1}^{\$} \\
&\quad -(1+i_0^{\$,syn})D_{F,0}^{\$,syn} - (1+i^{\$})D_{F,0}^{\$} - (1+i^{\$,swap})D_{swap,0}^{\$} + Q_{F,1}^{\$,spot}
\end{aligned} \tag{B.10}
$$

This shows how their dollar money balances at the end of period 1 are determined. Foreign households start with $M_{F,0}^{\$}$ dollar money balances from period 0. Then they receive $Y_{F,1}^{\$}$ dollars from exports. They need to make a dollar payment of $C_{FH,1}$ for buying US goods invoiced in dollars. Their dollar balances increase in period 1 due to borrowing $D_{F,1}^{\$,syn}$ dollars synthetically, $D_{F,1}^{\$}$ from US households and $D_{swap,1}^{\$}$ from the ECB. They drop due to principal and interest payments on all three types of dollar borrowing. Finally, $Q_{F,1}^{\$,spot}$ are dollars purchased in the spot market in period 1 in exchange for euros.

It follows that the spot market purchases by European households are ($dX = X_1 - X_0$)

$$
\begin{aligned}
Q_{F,1}^{\$,spot} &= dM_{F,1}^{\$} - Y_{F,1}^{\$} + C_{FH,1} - D_{F,1}^{\$,syn} - D_{F,1}^{\$} - D_{swap,1}^{\$} \\
&\quad +(1+i_0^{\$,syn})D_{F,0}^{\$,syn} + (1+i^{\$})D_{F,0}^{\$} + (1+i^{\$,swap})D_{swap,0}^{\$}
\end{aligned} \tag{B.11}
$$

we can also write this as

$$
\begin{aligned}
Q_{F,1}^{\$,spot} &= dM_{F,1}^{\$} - Y_{F,1}^{\$} + C_{FH,1} - dD_{F,1}^{\$,syn} - dD_{F,1}^{\$} - dD_{swap,1}^{\$} \\
&\quad +i_0^{\$,syn}D_{F,0}^{\$,syn} + i^{\$}D_{F,0}^{\$} + i^{\$,swap}D_{swap,0}^{\$}
\end{aligned} \tag{B.12}
$$

This in turn can be written as

$$
Q_{F,1}^{\$,spot} = dN_{F,1}^{\$} - Y_{F,1}^{\$} + C_{FH,1} + i_0^{\$,syn}D_{F,0}^{\$,syn} + i^{\$}D_{F,0}^{\$} + i^{\$,swap}D_{swap,0}^{\$} \tag{B.13}
$$

We do not need to consider US households as they only hold dollar bonds, receive dollars from exports and make payments in dollars for imports. They do not hold euro money balances.

B.2 CIP and UIP Arbitrageurs

The remaining agents are the CIP and UIP arbitrageurs. They are US agents. CIP arbitrageurs do not enter the spot market. Their profits in period 1 from their period 0 positions are denominated in dollars, while all the goods they consume are invoiced in dollars. They borrow dollars in period 1, but they lend these same dollars synthetically. They do not take any unhedged FX positions.

The US UIP arbitrageurs will enter the spot market. They receive $(1+i^{€})B^{€}_{UIP,0}$ euros from their time 0 euro position. They will also buy $B^{€}_{UIP,1}$ new euro bonds. They therefore sell $(1+i^{€})B^{€}_{UIP,0} - B^{€}_{UIP,1}$ euros, which is equivalent to buying

$$Q^{\$,spot}_{UIP,1} = -S_1 dB^{€}_{UIP,1} + S_1 i^{€} B^{€}_{UIP,0}$$

dollars.

The ECB also does not enter the spot market equilibrium as they simply pass the interest and principal payments on dollar loans to European households on to the Fed. The Fed does not enter the spot market equilibrium either as it does not pay interest on the euros that it receives from the ECB as part of the swap.

B.3 Equilibrium

Spot market equilibrium is

$$Q^{\$,spot}_{F,1} + Q^{\$,spot}_{UIP,1} = 0 \tag{B.14}$$

Substituting the expressions above gives

$$\begin{aligned} &dM^{\$}_{F,1} - Y^{\$}_{F,1} + C_{FH,1} - dD^{\$,syn}_{F,1} - dD^{\$}_{F,1} - dD^{\$}_{swap,1} + \\ &i^{\$,syn}_0 D^{\$,syn}_{F,0} + i^{\$} D^{\$}_{F,0} + i^{\$,swap} D^{\$}_{swap,0} - S_1 dB^{€}_{UIP,1} + S_1 i^{€} B^{€}_{UIP,0} = 0 \end{aligned} \tag{B.15}$$

The US trade account is equal to

$$TA^{\$}_{H,1} = Y_{H,1} - P_1 C_{H,1} - P_1 C^{o}_{H,1} \tag{B.16}$$

We also have

$$Y_{H,1} = C_{HH,1} + C_{FH,1} + C^{o}_{HH,1} \tag{B.17}$$

$$P_1 C_{H,1} = C_{HH,1} + C_{HF,1} \tag{B.18}$$

$$P_1 C^{o}_{H,1} = C^{o}_{HH,1} + C^{o}_{HF,1} \tag{B.19}$$

Therefore

$$TA^{\$}_{H,1} = C_{FH,1} - C_{HF,1} - C^{o}_{HF,1} \tag{B.20}$$

It follows that

$$-Y^{\$}_{F,1} + C_{FH,1} = -C_{HF,1} - C^{o}_{HF,1} + C_{FH,1} = TA^{\$}_{H,1} \tag{B.21}$$

We can then write the spot market equilibrium as

$$dM^{\$}_{F,1} + TA^{\$}_{H,1} - dD^{\$,syn}_{F,1} - dD^{\$}_{F,1} - dD^{\$}_{swap,1} \tag{B.22}$$
$$+i_0^{\$,syn} D^{\$,syn}_{F,0} + i^{\$} D^{\$}_{F,0} + i^{\$,swap} D^{\$}_{swap,0} - S_1 dB^{€}_{UIP,1} + S_1 i^{€} B^{€}_{UIP,0} = 0$$

We can write net factor income as

$$i_0^{\$,syn} B^{\$}_{CIP,0} + i^{\$} D^{\$}_{F,0} + i^{\$,swap} D^{\$}_{swap,0} + S_1 i^{€} B^{€}_{UIP,0} \tag{B.23}$$

The first term captures interest income earned by US CIP arbitrageurs for lending synthetic dollars to Europe. The second term is the interest earned by US households from lending dollars to European households. The third term is the interest that the Fed earns on the central bank swap line. The last term is interest paid to US UIP arbitrageurs on euro bonds. Using that $B^{\$}_{CIP,0} = D^{\$,syn}_{F,0}$, we then have

$$CA^{\$}_{H,1} = TA^{\$}_{H,1} + i_0^{\$,syn} D^{\$,syn}_{F,0} + i^{\$} D^{\$}_{F,0} + i^{\$,swap} D^{\$}_{swap,0} + S_1 i^{€} B^{€}_{UIP,0} \tag{B.24}$$

We can then write the spot market equilibrium as

$$CA^{\$}_{H,1} + dM^{\$}_{F,1} - dD^{\$,syn}_{F,1} - dD^{\$}_{F,1} - dD^{\$}_{swap,1} - S_1 dB^{€}_{UIP,1} = 0 \tag{B.25}$$

or

$$CA^{\$}_{H,1} + dN^{\$}_{F,1} - S_1 dB^{€}_{UIP,1} = 0 \tag{B.26}$$

Since $D^{\$,syn}_{F,1} = B^{\$}_{CIP,1}$, we can also write (B.25) as

$$CA^{\$}_{H,1} + dM^{\$}_{F,1} - dB^{\$}_{CIP,1} - dD^{\$}_{F,1} - dD^{\$}_{swap,1} - S_1 dB^{€}_{UIP,1} = 0 \tag{B.27}$$

or

$$CA^{\$}_{H,1} = dB^{\$}_{CIP,1} + dD^{\$}_{F,1} + dD^{\$}_{swap,1} + S_1 dB^{€}_{UIP,1} - dM^{\$}_{F,1} \tag{B.28}$$

This says that the current account is equal to net capital outflows. The first four terms on the right are capital outflows. The first term involves US CIP arbitrageurs lending synthetic dollars to Europe. The second term involves US households lending dollars to Europe. The third term involves the Fed lending dollars to Europe through a swap line. The fourth term involves US UIP arbitrageurs purchasing euro bonds. The last term is a US capital inflow in the form of European households acquiring dollar money balances in the US.

C Pre-Shock Equilibrium

Period 1 variables are equal to period 0 variables. For the exchange rate this implies $s_1 = s_0 = 0$. This also implies that $P_1 = P_1^* = 1$. Consumption is smoothed in that period 1 consumption by households is equal to period 2 consumption when $s_2 = 0$. We denote pre-shock period 1 variables with a bar. They are equal to corresponding period 0 variables.

In the pre-shock equilibrium saving of Home and Foreign households is zero, so that wealth is the same in period 1 as in period 0. This implies

$$\bar{C}_{H,1} = \bar{Y}_{H,1} + \bar{\Pi}_{HCB,1} + i^{\$} W_{H,0} \tag{C.29}$$

$$\bar{C}_{F,1} = \bar{Y}_{F,1} + \bar{\Pi}_{FCB,1} - i_0^{\$,syn} D_{F,0}^{\$,syn} - i^{\$} D_{F,0}^{\$} - i^{\$,swap} D_{swap,0}^{\$} + i^{€} B_{F,0}^{€} \tag{C.30}$$

This sets period 1 consumption equal to income, which is the sum of income from production and interest income and transfers of central bank profits back to the households. Here $\bar{\Pi}_{HCB,1} = i^{\$} M_0^{\$}$ and $\bar{\Pi}_{FCB,1} = i^{€} M_0^{€}$. One of these equations is redundant as aggregate world saving is zero. So we remove the last equation.

We have consumption smoothing in that period 1 consumption is equal to period 2 consumption when $s_2 = 0$. Then

$$\bar{C}_{H,1} = 1 + \bar{\Pi}_{HCB,2} + \bar{M}_{H,1}^{\$} + (1 + i^{\$}) \bar{W}_{H,1} \tag{C.31}$$

$$\begin{aligned} \bar{C}_{F,1} &= 1 + \bar{\Pi}_{FCB,2} + \bar{M}_{F,1}^{\$} + \bar{M}_{F,1}^{€} + (1 + i^{€}) \bar{W}_{F,1} \\ &\quad -(\bar{i}_1^{\$,syn} - i^{€}) \bar{D}_{F,1}^{\$,syn} - (i^{\$} - i^{€}) \bar{D}_{F,1}^{\$} - (i^{\$,swap} - i^{€}) \bar{D}_{swap,1}^{\$} \end{aligned} \tag{C.32}$$

The last two equations needed to derive the pre-shock equilibrium are

$$\bar{C}_{H,1} + \bar{C}_{F,1} + \bar{C}_{H,1}^{o} = 2 \tag{C.33}$$

$$\bar{B}_{CIP,1}^{\$} = \bar{D}_{F,1}^{\$,syn} \tag{C.34}$$

These correspond to the period 2 world goods market equilibrium, replacing $\bar{C}_{h,2} = \bar{C}_{h,1}$, and the period 1 swap market equilibrium. We then have a total of 5 equations: (C.29) and (C.31)-(C.34). This system can be solved by substituting expressions for money balances, portfolio holdings, central bank profits and period 1 production, setting $\bar{i}_1^{\$,syn} = i_0^{\$,syn}$, $s_1 = s_0 = 0$, $\bar{D}_{F,1}^{\$} = D_{F,0}^{\$}$, $\bar{D}_{swap,1}^{\$} = D_{swap,0}^{\$}$ and $\bar{W}_{h,1} = W_{h,0}$. We then have 5 equations in 5 variables: the 2 period 1 consumption levels, the 2 initial wealth levels $W_{h,0}$ and $\overline{cip}$. The interest differential i^d is exogenous.

Start with expressions for consumption of the "other agents":

$$\bar{C}^o_{H,1} = \frac{\overline{cip}^2}{\Gamma_{CIP}} + \frac{\left(i^d\right)^2}{\Gamma_{UIP}} \tag{C.35}$$

These are the consumption by respectively CIP and UIP arbitrageurs. We have

$$\bar{Y}_{H,1} = \bar{C}_{HH,1} + \bar{C}_{FH,1} + \bar{C}^o_{HH,1} = (1-\omega)\left(\bar{C}_{H,1} + \bar{C}^0_{H,1}\right) + \omega\bar{C}_{F,1} \tag{C.36}$$

Regarding central bank profits in the pre-shock equilibrium, we have

$$\bar{\Pi}_{HCB,2} = \bar{\Pi}_{HCB,1} = \psi i^{\$}\left(\bar{C}_{H,1} + \omega\bar{C}_{F,1}\right) \tag{C.37}$$

$$\bar{\Pi}_{FCB,2} = \bar{\Pi}_{FCB,1} = \psi i^{€}(1-\omega)\bar{C}_{F,1} \tag{C.38}$$

This uses that central bank bond holdings are equal to the money supply, which is equal to money demand.

Money demand expressions are

$$\bar{M}^{\$}_{H,1} = \psi\bar{C}_{H,1} \tag{C.39}$$

$$\bar{M}^{\$}_{F,1} = \psi\omega\bar{C}_{F,1} \tag{C.40}$$

$$\bar{M}^{€}_{F,1} = \psi(1-\omega)\bar{C}_{F,1} \tag{C.41}$$

Substituting these expressions into the 5 equations, we have

$$\omega\bar{C}_{H,1} = \omega\bar{C}_{F,1} + (1-\omega)\left[\frac{\overline{cip}^2}{\Gamma_{CIP}} + \frac{\left(i^d\right)^2}{\Gamma_{UIP}}\right] + \psi i^{\$}\left(\bar{C}_{H,1} + \omega\bar{C}_{F,1}\right) + i^{\$}W_{H,0} \tag{C.42}$$

$$(1-\psi)\bar{C}_{H,1} = 1 + \psi i^{\$}\left(\bar{C}_{H,1} + \omega\bar{C}_{F,1}\right) + (1+i^{\$})W_{H,0} \tag{C.43}$$

$$(1-\psi)\bar{C}_{F,1} = 1 + \psi i^{€}(1-\omega)\bar{C}_{F,1} + (1+i^{€})W_{F,0} - \frac{\overline{cip}(\overline{cip} + i^d)}{\Gamma_{CIP}} - i^d\bar{D}^{\$}_F$$
$$-(i^{\$,swap} - i^{€})\bar{D}^{\$}_{swap} \tag{C.44}$$

$$\bar{C}_{H,1} + \bar{C}_{F,1} + \frac{\overline{cip}^2}{\Gamma_{CIP}} + \frac{\left(i^d\right)^2}{\Gamma_{UIP}} = 2 \tag{C.45}$$

$$\rho + \psi\omega\bar{C}_{F,1} - \bar{D}^{\$}_F - \bar{D}^{\$}_{swap} - \frac{\overline{cip} + i^d}{\Gamma_F} = \frac{\overline{cip}}{\Gamma_{CIP}} \tag{C.46}$$

D Noise Trader Shocks

Here we show that when adding noise traders to the model, we have another shock that only shifts the spot market schedule. Noise traders choose exogenous portfolios of dollar and euro bonds, with $B^{\$}_{noise,1} + S_1 B^{€}_{noise,1} = 0$. Let $B^{€}_{noise,1} = -n_1$, so that (after linearization around $n_1 = 0$) $B^{\$}_{noise,1} = n_1$. An increase in n_1 therefore implies an exogenous portfolio shift from euro to dollar bonds.

We then need to add noise traders to the spot market equilibrium. We have

$$Q^{\$,spot}_{noise,1} = n_1 \tag{D.47}$$

The linearized spot market equilibrium will remain the same, but with the additional term in the expression of $shock^{spot}_1$ equal to $-\Gamma_F \hat{n}_1$.