Online Appendix

Foreign Exchange Intervention with UIP and CIP Deviations

Philippe Bacchetta, Kenza Benhima, and Brendan Berthold

June 2026

A CHF and JPY as Safe-Haven Currencies

In this Appendix, we show evidence on safe haven properties of the CHF and JPY. The UIP deviation should be more negative in safe-haven economies when global risk increases. We examine this property in this appendix. Table A.1 shows the estimated impact of global risk on the UIP deviations ($\mathbb{E}X^*$) for the CHF and JPY, as well as for other currencies, using different measures of risk. Other currencies include EUR, GBP, AUD, CAD, SEK, NOK, NZD, and DKK. We compute UIP deviations using short-term rates from Datastream and survey data from Consensus Economics. The risk measures are the Global Economic Policy Uncertainty (GEPU) index developed in Baker et al. (2016), the US EPU (USEPU), and the World Uncertainty Index (WUI) developed in Ahir et al. (2022). We use OLS regressions with country fixed effects and with heterogeneous dynamics, controlling for the lagged risk measure and the lagged UIP deviation. We report only the coefficient associated with the contemporaneous risk measure.

When using GEPU and USEPU as risk measures, the estimated impact of risk on CHF and JPY UIP deviations is significantly negative, while it is not significant for other currencies. This is consistent with the safe-haven role of the CHF and the JPY, in contrast to other major currencies. Agents demand these two currencies as a hedge against global uncertainty, thereby consenting to lower expected returns. When using WUI, results are not significant. This might be explained by the fact that WUI is at the quarterly frequency, which does not allow for as precise an identification as with monthly data.

Table A.1: Impact of risk on UIP deviations (EX*)

	GEPU	USEPU	WUI
Switzerland (CHF)	-0.0213^{**}	-0.0167^{***}	-0.0036
	(0.0100)	(0.0063)	(0.0075)
Japan (JPY)	-0.0245^{**}	-0.0227^{***}	0.0030
	(0.0123)	(0.0062)	(0.0084)
Others	0.0008	0.0017	0.0003
	(0.0041)	(0.0023)	(0.0025)

Notes: This table displays OLS estimates of the UIP deviation (EX*) at a 3-month horizon on a standardized risk measure interacted with three currency groups: CHF, JPY, and others, following the specification: $\mathbb{E}_t(X^*_{i,t+3}) = \alpha^i_0 + (\beta_{CHF} \cdot 1_{i=CHF} + \beta_{JPY} \cdot 1_{i=JPY} + \beta_{other} \cdot 1_{i=other}) \cdot risk\_measure_t + \alpha^i_1 \cdot risk\_measure_{t-1} + \alpha^i_2 \cdot \mathbb{E}_{t-1}(X^*_{i,t+2}) + u_{i,t}$. α^i_0 are currency fixed effects, and α^i_1 and α^i_2 are heterogeneous currency-specific slope coefficients. β_{CHF}, β_{JPY} and β_{other} are the coefficients reported in the table. The sample period is 1999–2022. Regressions using GEPU and USEPU are at the monthly frequency. Since WUI is only available at a quarterly frequency, we take the quarterly mean of UIP deviations and run a quarterly regression. HC1 robust standard errors in parentheses. All risk variables are standardized over the 1999–2022 sample. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

B Computing excess returns and stochastic discount factors for financial intermediaries

In this section, we discuss the construction of $cov(x^*_{t+1}, m^*_{t+1})/\mathbb{E}_t m^*_{t+1}$, considering either the CHF or the JPY as the domestic currency, and keeping the USD as the foreign currency.

B.1 Excess returns

For i_t, we rely on the domestic (CHF or JPY) 3-month risk-free rate, while i^*_t is the US 3-month risk-free rate. For s_t we use nominal spot exchange rate data expressed in the amount of domestic currency per unit of USD. All data is from Datastream and is retrieved at the daily frequency. The daily data are aggregated to the monthly or quarterly frequency by taking the mean within each quarter. To compute excess returns, we first compute quarterly excess returns according to (31). We assume that what matters for financial intermediaries is the moving excess returns of this carry-trade over the past year by taking a moving sum of excess returns over that of the current and last three quarters. This allows us to have a smoother version of excess returns.

B.2 Stochastic discount factors

We now discuss the construction of the SDF of financial intermediaries, which is defined as $m^* = \beta \left(NW_{t+1}/NW_t\right)^{-\gamma}$. Similarly to He et al. (2017), we define $NW_{t+1} = \eta_{t+1} \times W_{t+1}$, where η_{t+1} is a measure of the capital ratio of financial intermediaries and W_t is a measure of total wealth. The SDF is obtained by interacting a measure of the growth rate of the capital ratio and total wealth. We discuss below the construction of these growth rates.

We consider two measures of the capital ratio. The first specification (HKM) relies on the capital ratio measure from He et al. (2017) which is retrieved from Zhiguo He's website at a daily frequency and aggregated at a monthly or quarterly frequency by taking the mean. The second specification (AEM) is based on Adrian et al. (2014) and is computed using quarterly balance sheet data from the Federal Reserve Flow Of Funds (Table L.130). To obtain the capital ratio innovation, we regress the one-year growth in the capital ratio on its current value, and define the capital risk factor as the resulting residual divided by the current capital ratio. The two resulting measures are defined as $\hat{\eta}_{t+1}^{\text{HKM}}$ and $\hat{\eta}_{t+1}^{\text{AEM}}$, respectively.

For total wealth growth, we rely on a financial measure (MSCI World Equity Index) and a real measure (US GDP). For the financial measure, we compute the yearly return on the MSCI World Equity Index in excess of the US risk-free rate. The resulting series is defined as ΔW_{t+1}^{MSCI}. For the real measure, we compute the year-over-year log growth rate of US GDP. The resulting series is defined as ΔW_{t+1}^{GDP}.

The SDF of financial intermediaries is then computed as $m_{t+1}^{*} = \beta((1+\Delta\eta_{t+1}^{i}) \times (1+\Delta W_{t+1}^{j}))^{-\gamma}$ for $i \in \{AEM, HKM\}$ and $j \in \{MSCI, GDP\}$, with $\beta = 0.99$ and $\gamma = 10$. This gives rise to 4 potential specifications of the SDF of financial intermediaries. To confirm the suitability of our specification of the SDF, we perform formal cross-sectional two-step asset pricing tests using the approach of He et al. (2017) and confirm the ability of our SDF specification to explain returns in currency portfolios.

B.3 Implications for b^{max}

Here we relate a lower bound on b^{max} to moments in the data, using, among others, our estimates of the wealth of financial intermediaries.

Proposition 1 *b^{max} admits a lower bound: $b^{max} \geq \underline{b}^{max}$. With CRRA and a coefficient of relative risk aversion γ, this lower bound is*[1]

$$\underline{b}^{max} = \frac{\gamma\frac{V(\tilde{y}^*)-Cov(\tilde{y}^*,\tilde{y})}{V(\tilde{y}^*)}Cov(\tilde{y}^*,\tilde{S}_2) - CIP}{CIP/b^{H*} + \gamma\frac{Cov(\tilde{y}^*,\tilde{S}_2)^2}{V(\tilde{y}^*)}} - \frac{Cov(\tilde{y}^*,\tilde{y})b^G}{Cov(\tilde{y}^*,\tilde{S}_2)} \tag{B.1}$$

Remember, $b^{max} = \frac{(1-\alpha)\rho\sigma^2-\chi}{\Gamma+\rho^2\sigma^2} - \frac{\alpha\rho\sigma^2\min(nfl^{opt},b^G)}{\Gamma+\rho^2\sigma^2}$ in the log-utility case that we use in the theoretical analysis. With CRRA utility, this becomes $b^{max} = \frac{\gamma(1-\alpha)\rho\sigma^2-\chi}{\Gamma+\gamma\rho^2\sigma^2} - \frac{\gamma\alpha\rho\sigma^2\min(nfl^{opt},b^G)}{\Gamma+\gamma\rho^2\sigma^2}$, where γ is the coefficient of relative risk aversion.

---

[1] In a two-period model, this lower bound is the same under CRRA utility and Epstein-Zin utility. Indeed, the condition $\mathbb{E}(mX^*) = 0$ that defines b^{max} is the same, and it only depends on the coefficient of relative risk aversion. Indeed, we can show that the stochastic discount factor m is equal to $c_2^{-\gamma}$, up to a term μ_1 known in period 1. Substituting $\mu_1 c_2^{-\gamma}$ to c_2^{-1} in our derivations, we would obtain the same condition on b^{max}.

First, notice that $\min(nfl^{opt}, b^G) \leq b^G$. As a consequence,

$$b^{max} \geq \frac{\gamma(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \gamma\rho^2\sigma^2} - \frac{\gamma\alpha\rho\sigma^2 b^G}{\Gamma + \gamma\rho^2\sigma^2}$$

Now, remember that $CIP = \Gamma b^{H*} + \chi$. Consequently, $\Gamma \leq CIP/b^{H*}$ and $\chi \leq CIP$. Additionally, $\Gamma \geq 0$ by assumption. Therefore,

$$b^{max} \geq \frac{\gamma(1-\alpha)\rho\sigma^2 - CIP}{CIP/b^{H*} + \gamma\rho^2\sigma^2} - \frac{\gamma\alpha\rho\sigma^2 b^G}{\gamma\rho^2\sigma^2} = \frac{\gamma(1-\alpha)\rho\sigma^2 - CIP}{CIP/b^{H*} + \gamma\rho^2\sigma^2} - \frac{\alpha b^G}{\rho} = \underline{b}^{max}$$

Finally, notice that $V(\tilde{y}^*) = \sigma^2$, $Cov(\tilde{y}^*, \tilde{y}) = \alpha\sigma^2$, $Cov(\tilde{y}^*, \tilde{S}_2) = \rho\sigma^2$. We can then fully relate the lower bound $\underline{b}^{max}$ to the data, as described in Equation (B.1).

Table B.1 provides estimates of the lower bound $\underline{b}^{max}$, for both Switzerland and Japan, using estimates of the moments $V(\tilde{y}^*)$, $Cov(\tilde{y}^*, \tilde{y})$, $Cov(\tilde{y}^*, \tilde{S}_2)$, the CIP deviation, b^{H*} and b^G from the data for the period 2010-2022, and with $\gamma = 10$ as we assumed to produce Table 1. We estimate $V(\tilde{y}^*)$ as the variance of the growth rate of the wealth of financial intermediaries, computed as $\Delta\tilde{y}^*_{t+1} = (1 + \Delta\eta^i_{t+1}) \times (1 + \Delta W^j_{t+1}) - 1$ for $i \in \{HKM, AEM\}$ and $j \in \{MSCI, GDP\}$. We estimate $Cov(\tilde{y}^*, \tilde{y})$ as the covariance between the growth rate of the wealth of financial intermediaries $\Delta\tilde{y}^*_{t+1}$ and the growth rate of domestic GDP $\Delta\tilde{y}_{t+1}$. We estimate $Cov(\tilde{y}^*, \tilde{S}_2)$ as the covariance between the growth rate of the wealth of financial intermediaries $\Delta\tilde{y}^*_{t+1}$ and the annualized quarterly depreciation rate of the exchange rate $\Delta\tilde{S}_{t+1}$. The CIP is measured as described in Appendix **??**. b^G is measured as the average over the period 2010-2022 of the general government debt as a percentage of GDP (IMF). We set it at 41% for Switzerland and 233% for Japan. b^{H*} is the average over 2010-2022 of the sum of the general government debt and official foreign reserves, as a percentage of GDP (IMF). The latter is set at 137% of GDP for Switzerland, and 258% for Japan.

The bounds implied by these figures and Equation (B.1) are high and well above the estimated supply of gross foreign liabilities b^{H*} for most measures (all but one).

C Robustness checks for household utility

This appendix presents robustness checks for the household covariance estimates reported in Table 1. Since the household SDF is unobserved, we document that the

	CHF	JPY	CHF	JPY	CHF	JPY	CHF	JPY
CIP	0.24%	0.28%	-	-	-	-	-	-
b^{H*}	137%	258%	-	-	-	-	-	-
b^{G}	41%	233%	-	-	-	-	-	-
NW_{t+1}	$\eta^{HKM}_{t+1} \times W^{MSCI}_{t+1}$		$\eta^{HKM}_{t+1} \times W^{GDP}_{t+1}$		$\eta^{AEM}_{t+1} \times W^{MSCI}_{t+1}$		$\eta^{AEM}_{t+1} \times W^{GDP}_{t+1}$	
$V(\Delta\tilde{y}^{*}_{t+1})$	0.30%	-	2.26%	-	4.68%	-	7.36%	-
$\text{Cov}(\Delta\tilde{y}^{*}_{t+1}, \Delta\tilde{y}_{t+1})$	0.03%	0.03%	-0.07%	-0.01%	0.27%	0.28%	0.19%	0.27%
$\text{Cov}(\Delta\tilde{y}^{*}_{t+1}, \Delta\tilde{S}_{t+1})$	0.09%	0.27%	0.13%	0.16%	0.27%	0.67%	0.20%	0.80%
$\underline{b}^{max}$	115%	53%	470%	609%	661%	464%	723%	679%

Table B.1: Estimated lower bound for b^{max} (2010-2022)

Note: This table provides the estimates of $V(\Delta\tilde{y}^{*}_{t+1})$, $\text{Cov}(\Delta\tilde{y}^{*}_{t+1}, \Delta\tilde{y}_{t+1})$, $\text{Cov}(\Delta\tilde{y}^{*}_{t+1}, \Delta\tilde{S}_{t+1})$, using the proxies of the wealth of (international) financial intermediaries. Values are expressed in percentage points, over the period 2010-2022. It also provides the averages of the CIP, of b^{H*} and b^{G} over that period.

key finding, namely that the covariance of the financial intermediary SDF with FX excess returns substantially exceeds that of the household SDF, is robust across three distinct preference specifications and a wide range of economically plausible parameter values. We consider the following utility specifications: constant relative risk aversion utility (CRRA), recursive preferences with separable elasticity of intertemporal substitution and risk aversion (Epstein-Zin), and time-varying effective risk aversion driven by external habit (Campbell and Cochrane (1999)).

C.1 Utility specifications

CRRA. The baseline household utility specification is constant relative risk aversion (CRRA). The SDF is:

$$m_{t+1} = \beta\, e^{-\gamma \Delta c_{t+1}}, \tag{C.1}$$

where $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$ is the quarterly log growth rate of real per-capita consumption and $\beta = 0.99$. We consider $\gamma \in \{2,\ 5,\ 10,\ 20\}$.

Epstein-Zin. To allow the elasticity of intertemporal substitution (EIS) to differ from the inverse of risk aversion, we consider Epstein-Zin recursive preferences.

The agent has recursive utility over consumption streams:

$$V_t = \left[(1-\beta)\, C_t^{1-1/\psi} + \beta \left(\mathbb{E}_t\big[V_{t+1}^{1-\gamma} \big] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}, \tag{C.2}$$

where γ is the coefficient of relative risk aversion, ψ is the EIS, and $\beta = 0.99$. When $\gamma = 1/\psi$ these preferences collapse to CRRA. Defining $\theta \equiv (1-\gamma)/(1-1/\psi)$, the implied SDF is:

$$m_{t+1} = \beta^{\theta}\, e^{-(\theta/\psi)\Delta c_{t+1}}\, R_{w,t+1}^{\theta-1}, \tag{C.3}$$

where $R_{w,t+1}$ is the gross domestic real equity return, deflated by the domestic CPI. We consider $\gamma \in \{2, 5, 10, 20\}$ and $\psi \in \{1.5, 2.0, 2.5\}$, giving a 12-point parameter grid.

Campbell-Cochrane habit formation. To capture time-varying effective risk aversion, we use the external habit framework of Campbell and Cochrane (1999). The agent maximises:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}, \tag{C.4}$$

where X_t is an external habit level. Defining the surplus consumption ratio $S_t \equiv (C_t - X_t)/C_t \in (0,1)$, effective risk aversion is γ/S_t, which rises as consumption approaches habit. The SDF is:

$$m_{t+1} = \delta \left(\frac{S_{t+1}}{S_t} \cdot \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{C.5}$$

The log surplus ratio $s_t \equiv \log S_t$ evolves as:

$$s_{t+1} = (1-\phi)\,\bar{s} + \phi\, s_t + \lambda(s_t)\,(\Delta c_{t+1} - \bar{g}_c), \tag{C.6}$$

where $\bar{s} = \log \bar{S}$ and $\bar{g}_c$ is the sample mean of log consumption growth. The sensitivity function:

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1 & \text{if } s_t < s_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{C.7}$$

is chosen so that the risk-free rate is exactly constant (Campbell and Cochrane, 1999). The steady-state surplus ratio is $\bar{S} = \sigma_c\sqrt{\gamma/(1-\phi)}$, where σ_c is the sample standard deviation of log per-capita consumption growth. The maximum log surplus is $s_{\max} = \log \bar{S} + \frac{1}{2}(1 - \bar{S}^2)$, beyond which habit does not respond to consumption shocks. All parameters ($\bar{S}$, $\bar{g}_c$, σ_c) are estimated separately from each country's consumption series. We use $\phi = 0.87^{1/4}$ and $\delta = 0.99^{1/4}$, based on Campbell and Cochrane (1999) calibration, and consider $\gamma \in \{1, 2, 5\}$.

C.2 Robustness checks

CRRA. Table C.1 reports normalized covariances for the CRRA household SDF across $\gamma \in \{2, 5, 10, 20\}$. For the post-2010 subsample, covariances are positive but small for both CHF and JPY across the entire parameter grid, and statistically insignificant. The maximum post-2010 value (0.486% for JPY at $\gamma = 20$) is well below that of the financial intermediary benchmark. This reflects the well-known smoothness of per-capita consumption growth, which generates insufficient covariance with FX excess returns.

Table C.1: Household CRRA SDF: Sensitivity to Risk Aversion γ. Normalized covariance $\text{Cov}(x^*_{t+1}, m_{t+1})/\mathbb{E}_t m_{t+1}$.

	2010–2022	
γ	CHF/USD	JPY/USD
2	0.007%	0.032%
5	0.023%	0.087%
10	0.068%	0.194%
20	0.217%	0.486%

Notes: SDF formula: $m_{t+1} = \beta e^{-\gamma \Delta c_{t+1}}$, $\beta = 0.99$. Consumption: quarterly per-capita real consumption for Switzerland (CHF) and Japan (JPY). Sample: 2010–2022. ***, **, * denote significance at 1%, 5%, 10% (Newey-West).

Epstein-Zin. Table C.2 reports results for the Epstein-Zin SDF across $\gamma \in \{2, 5, 10, 20\}$ and $\psi \in \{1.5, 2.0, 2.5\}$. For the post-2010 subsample, Epstein-Zin preferences generate somewhat larger covariances than CRRA for the CHF at $\gamma = 20$ and $\psi = 1.5$, reaching 1.69% though not statistically significant. Covariances post 2010 for the JPY are negative across the board. Overall, the results confirm that the covariance between FX excess returns and the household SDF remains statistically insignificant or negative post-2010 across the entire Epstein-Zin parameter grid, for both currencies.

Table C.2: Household Epstein-Zin SDF: Sensitivity to (γ, ψ). Normalized covariance $\text{Cov}(x^*_{t+1}, m_{t+1})/\mathbb{E}_t m_{t+1}$.

		2010–2022	
γ	ψ	CHF/USD	JPY/USD
2	1.5	0.026%	−0.153%
	2.0	0.024%	−0.104%
	2.5	0.023%	−0.088%
5	1.5	0.053%	−0.753%
	2.0	0.032%	−0.403%
	2.5	0.032%	−0.310%
10	1.5	0.515%	−2.595%
	2.0	0.158%	−1.239%
	2.5	0.090%	−0.879%
20	1.5	1.690%	−5.871%
	2.0	0.911%	−2.892%
	2.5	0.629%	−1.978%

Notes: $m_{t+1} = \beta^\theta e^{-(\theta/\psi)\Delta c_{t+1}} R^{\theta-1}_{w,t+1}$, $\theta = (1-\gamma)/(1-1/\psi)$; $R_{w,t+1}$ is the domestic real equity return (deflated by domestic CPI), $\beta = 0.99$. Sample: 2010–2022. ***, **, * denote significance at 1%, 5%, 10% (Newey-West).

Campbell-Cochrane habit formation. Table C.3 reports results for the Campbell-Cochrane utility function (Campbell and Cochrane, 1999) across $\gamma \in \{1, 2, 5\}$. The time-varying effective risk aversion mechanism γ/S_t amplifies SDF volatility relative to CRRA, and this is reflected in larger post-2010 covariances: at $\gamma = 5$, which translates into an effective risk aversion of approximately 28 for CHF and 30 for JPY at the steady state, the Campbell-Cochrane specification generates 0.384% for CHF (not statistically significant) and 1.018% (statistically significant at 10%) for JPY. These post-2010 covariances are thus statistically insignificant for CHF across all parameter values, and significant only for JPY at $\gamma = 5$. This being said, they remain well below the HKM-based financial intermediary covariances.

Table C.3: Household Campbell-Cochrane SDF: Sensitivity to Risk Aversion γ. Normalized covariance $\text{Cov}(x^*_{t+1}, m_{t+1})/\mathbb{E}_t m_{t+1}$.

	Effective γ		2010–2022	
γ	CHF	JPY	CHF/USD	JPY/USD
1	12.5	13.3	0.030%	0.212%
2	17.7	18.8	0.130%	0.449%
5	28.0	29.7	0.384%	1.018%*

Notes: $m_{t+1} = \delta(S_{t+1}C_{t+1}/S_tC_t)^{-\gamma}$; S_t constructed following Campbell and Cochrane (1999) with $\phi = 0.87^{1/4}$, $\delta = 0.99^{1/4}$. Country-specific calibration of $(\bar{S}, \bar{g}_c, \sigma_c)$. Effective risk aversion is $\gamma/\bar{S}$, where $\bar{S} = \sigma_c\sqrt{\gamma/(1-\phi)}$ is the steady-state surplus ratio. Consumption: quarterly per-capita real. Sample: 2010–2022. ***, **, * denote significance at 1%, 5%, 10% (Newey-West).

D Decentralized Model Properties

In this section, we derive some key properties of the decentralized model. We first develop a preliminary analysis of asset prices (D.1), the households' stochastic discount factor (D.2), and the households' currency FOCs (D.3). Then we prove Proposition 1 (D.4), the impact of FX interventions on nfl (D.5), the expression for the covariance differential (28) (D.6), and Proposition 3 (D.7).

We use a second-order approximation to solve the model. We denote the variables in log with a tilde. For instance: $\tilde{y} = \log(y)$ and $\tilde{y}^* = \log(y^*)$. We also define

$\tilde{i}^* = \log(1+i^*)$ and $\tilde{i} = \log(1+i)$.

D.1 Asset Pricing by Foreign Intermediaries

In this sub-section, we derive the asset pricing equations. The foreign interest rate is set exogenously in a small open economy. We have assumed that $i^* = \beta^{-1} - 1$, which implies

$$\tilde{i}^* = -\log(\beta). \tag{D.1}$$

Equation (6) determines the domestic asset prices $\tilde{i} + \tilde{S}_1$:

$$\begin{aligned} \mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right) &= \mathbb{E}\left(m^*(1+i^*)\right) + \chi + \Gamma b^{H*} \\ \Leftrightarrow \qquad \mathbb{E}(e^{\tilde{m}^* - \tilde{S}_2 + \tilde{i} + \tilde{S}_1}) &= 1 + \chi + \Gamma b^{H*} \end{aligned} \tag{D.2}$$

where we used $\mathbb{E}\left(m^*(1+i^*)\right) = 1$.

$$\begin{aligned} \Leftrightarrow \qquad e^{\mathbb{E}(\tilde{m}^* - \tilde{S}_2) + \frac{1}{2}V(\tilde{m}^* - \tilde{S}_2) + \tilde{i} + \tilde{S}_1} &= 1 + \chi + \Gamma b^{H*} \\ \Leftrightarrow \qquad e^{\log(\beta) - \mathbb{E}((1+\rho)\tilde{y}^*) + \rho(1-\rho)\sigma^2/2 + \frac{1}{2}V((1+\rho)\tilde{y}^*) + \tilde{i} + \tilde{S}_1} &= 1 + \chi + \Gamma b^{H*} \\ \Leftrightarrow \qquad e^{\log(\beta) - (1+\rho)\sigma^2/2 + \rho(1-\rho)\sigma^2/2 + (1+\rho)^2\sigma^2/2 + \tilde{i} + \tilde{S}_1} &= 1 + \chi + \Gamma b^{H*} \\ \Leftrightarrow \qquad e^{\log(\beta) + \rho\sigma^2 + \tilde{i} + \tilde{S}_1} &= 1 + \chi + \Gamma b^{H*} \end{aligned}$$

where we used $\mathbb{E}(e^x) = e^{\mathbb{E}(x) + V(x)/2}$, $\tilde{m}^* = log(\beta) - \tilde{y}^*$, $\tilde{S}_2 = \rho[\tilde{y}^* - (1-\rho)\sigma^2/2]$ and $\tilde{y}^* \sim N(\sigma^2/2, \sigma^2)$. This implies

$$e^{\log(\beta) + \rho\sigma^2 + \tilde{i} + \tilde{S}_1} = 1 + \chi + \Gamma b^{H*}$$

which we linearly approximate to

$$\tilde{i} + \tilde{S}_1 = \chi + \Gamma b^{H*} - \rho\sigma^2 - \log(\beta). \tag{D.3}$$

This determines $\tilde{i} + \tilde{S}_1$ as a function of b^{H*}.

D.2 Households Stochastic Discount Factor

We derive here the households' stochastic discount factor $\tilde{m}$. To obtain $\tilde{m}$, we can rewrite the resource constraints (22) as

$$\begin{aligned} c_1 &= y_1\left(1 + \tfrac{nfl}{y_1}\right) \\ c_2 &= y_2\left(1 - \tfrac{nfl}{y_1}\tfrac{1+i^*}{1+g_2} - \tfrac{b^{H*}}{y_1}\tfrac{X^*}{1+g_2}\right) \end{aligned}$$

with $1+g_2 = y_2/y_1$. Taking logs and using a second-order approximation (assuming nfl/y_1, b^{H*}/y_1, X^* and g_2 are small), we obtain

$$\begin{aligned}
\tilde{c}_1 &= \tilde{y}_1 + \tfrac{nfl}{y_1} - \tfrac{1}{2}\left(\tfrac{nfl}{y_1}\right)^2 \\
\tilde{c}_2 &= \tilde{y}_2 - \tfrac{nfl}{y_1}(1+i^* - g_2) - \tfrac{1}{2}\left(\tfrac{nfl}{y_1}\right)^2 (1+i^*) - \tfrac{b^{H*}}{y_1}X^*
\end{aligned}$$

Finally, we use the approximation $g_2 = \tilde{y}_2 - \tilde{y}_1$ along with the assumption that $y_1 = 1$ and hence $\tilde{y}_1 = 0$ to obtain the approximated household's budget constraints:

$$\begin{aligned}
\tilde{c}_1 &= nfl - \frac{1}{2}nfl^2 \\
\tilde{c}_2 &= \tilde{y}_2\,(1+nfl) - \left(nfl + \frac{1}{2}nfl^2\right)(1+i^*) - b^{H*}X^*
\end{aligned} \tag{D.4}$$

Using (D.4), we get

$$\begin{aligned}
\tilde{m} &= \log(\beta) + \tilde{c}_1 - \tilde{c}_2 \\
&= \log(\beta) - \tilde{y}_2\,(1+nfl) + nfl(2+\tilde{i}^*) + \tilde{i}^* nfl^2/2 + b^{H*}(\tilde{i} - \tilde{i}^* + \tilde{S}_1 - \tilde{S}_2) \\
&= \log(\beta) - \alpha\tilde{y}^*\,(1+nfl) + nfl(2+\tilde{i}^*) + \tilde{i}^* nfl^2/2 + b^{H*}(\tilde{i} - \tilde{i}^* + \tilde{S}_1 - \rho\tilde{y}^*) \\
&\quad + [\alpha(1-\alpha)(1+nfl) + \rho(1-\rho)b^{H*}]\sigma^2/2 - g(1+nfl) \\
&= \log(\beta) - [\alpha\,(1+nfl) + \rho b^{H*}]\tilde{y}^* + nfl(2+\tilde{i}^*) + \tilde{i}^* nfl^2/2 + b^{H*}(\tilde{i} - \tilde{i}^* + \tilde{S}_1) \\
&\quad + [\alpha(1-\alpha)(1+nfl) + \rho(1-\rho)b^{H*}]\sigma^2/2 - g(1+nfl),
\end{aligned}$$

where we used $\tilde{y}_2 = g + \alpha\tilde{y}^* - \alpha(1-\alpha)\sigma^2/2$ and $\tilde{S}_2 = \rho\tilde{y}^* - \rho(1-\rho)\sigma^2/2$, and the approximation of X^*: $X^* = \tilde{i} - \tilde{i}^* + \tilde{S}_1 - \tilde{S}_2$.

Using The asset pricing equations (D.1) and (D.3), we thus have

$$\begin{aligned}
\tilde{m} &= \log(\beta) - [\alpha\,(1+nfl) + \rho b^{H*}]\tilde{y}^* + nfl(2 - \log(\beta)) - \log(\beta)nfl^2/2 + b^{H*}(\chi + \Gamma b^{H*} - \rho\sigma^2) \\
&\quad + [\alpha(1-\alpha)(1+nfl) + \rho(1-\rho)b^{H*}]\sigma^2/2 - g(1+nfl)
\end{aligned} \tag{D.5}$$

D.3 Households' Currency FOCs

We expand here the households' FOC with respect to nfl and b^{H*}. We use the household's FOCs (13) and (14), which we rewrite as follows

$$1 - \mathbb{E}\left(e^{\tilde{m}+\tilde{i}^*}\right) = \lambda^F$$

$$\mathbb{E}\left(e^{\tilde{m}+\tilde{i}^*} - e^{\tilde{m}+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right) = \lambda^H - \lambda^F$$

This system is equivalent to

$$
\begin{aligned}
\mathbb{E}\left(e^{\tilde{m}+\tilde{i}^*}\right) &= 1-\lambda^F \\
\frac{\mathbb{E}\left(e^{\tilde{m}+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}\left(e^{\tilde{m}+\tilde{i}^*}\right)} &= 1-\frac{\lambda^H-\lambda^F}{1-\lambda^F}
\end{aligned}
$$

Substituting for $\tilde{i}^*$ and $\tilde{i}+\tilde{S}_1$ using the asset pricing equations (D.1) and (D.3), and $\mathbb{E}(e^x)=e^{\mathbb{E}(x)+V(x)/2}$, we can rewrite these equations as follows:

$$
e^{\mathbb{E}(\tilde{m})+V(\tilde{m})/2-\log(\beta)} = 1-\lambda^F \tag{D.6}
$$

$$
\begin{aligned}
\frac{e^{\mathbb{E}(\tilde{m}-\tilde{S}_2)+V(\tilde{m}-\tilde{S}_2)/2+\chi+\Gamma b^{H*}-\rho\sigma^2-\log(\beta)}}{e^{\mathbb{E}(\tilde{m})+V(\tilde{m})/2-\log(\beta)}} &= \\
e^{-\mathbb{E}(\tilde{S}_2)+V(\tilde{S}_2)/2-Cov(\tilde{m},\tilde{S}_2)+\chi+\Gamma b^{H*}-\rho\sigma^2} &= \\
e^{-Cov(\tilde{m},\tilde{S}_2)+\chi+\Gamma b^{H*}-\rho\sigma^2} &= 1-\frac{\lambda^H-\lambda^F}{1-\lambda^F}
\end{aligned} \tag{D.7}
$$

where we used $-\mathbb{E}(\tilde{S}_2)+V(\tilde{S}_2)/2=0$. These two expressions depend on $\mathbb{E}(\tilde{m})+V(\tilde{m})/2$ and $Cov(\tilde{m},\tilde{S}_2)$. We use the expression for $\tilde{m}$ to write:

$$
\begin{aligned}
&\mathbb{E}(\tilde{m})+\tfrac{V(\tilde{m})}{2} = \\
&\log(\beta)+nfl(2-\log(\beta))-\log(\beta)nfl^2/2+b^{H*}(\chi+\Gamma b^{H*}-\rho\sigma^2)-g(1+nfl) \\
&+[\alpha^2(1+nfl)nfl+\rho^2 b^{H*}(b^{H*}-1)+2\alpha\rho(1+nfl)b^{H*}]\tfrac{\sigma^2}{2}
\end{aligned} \tag{D.8}
$$

and

$$
Cov(\tilde{m},\tilde{S}_2)=\rho Cov(\tilde{m},\tilde{y}^*)=-\rho[\alpha\,(1+nfl)+\rho b^{H*}]\sigma^2 \tag{D.9}
$$

Substituting for $\mathbb{E}(\tilde{m})+V(\tilde{m})/2$ and $Cov(\tilde{m},\tilde{S}_2)$ in (D.6) and (D.7) using (D.8) and (D.9), and taking logs, we obtain the following:

$$
\begin{aligned}
nfl(2-\log(\beta))-\log(\beta)nfl^2/2+b^{H*}(\chi+\Gamma b^{H*}-\rho\sigma^2)-g(1+nfl)& \\
+[\alpha^2(1+nfl)nfl+\rho^2 b^{H*}(b^{H*}-1)+2\alpha\rho(1+nfl)b^{H*}]\frac{\sigma^2}{2}& \\
=\log(1-\lambda^F)&
\end{aligned} \tag{D.10}
$$

$$
[\rho^2 b^{H*}+\alpha\rho(1+nfl)]\sigma^2+\chi+\Gamma b^{H*}-\rho\sigma^2=\log\left(1-\frac{\lambda^H-\lambda^F}{1-\lambda^F}\right) \tag{D.11}
$$

D.4 Proof of Proposition 1

To prove Proposition 1 (the effectiveness of FX interventions), it is useful to lay down some preliminary steps. We first characterize the solution that satisfies both intertemporal and portfolio optimality. We then characterize some important properties of nfl and b^{H*}. Finally, we derive our results.

D.4.1 Intertemporal and portfolio optimality

The following lemma characterizes intertemporal and portfolio optimality.

Lemma 2 (Intertemporal and portfolio optimality) *(i) For a given b^{H*}, the net foreign liabilities nfl that satisfy intertemporal optimality are characterized by*

$$nfl = nfl^{opt}(b^{H*}) \tag{D.12}$$

where $nfl^{opt}(b^{H*})$ satisfies $P[nfl^{opt}(b^{H*}), b^{H*}] = 0$ with

$$P[nfl, b^{H*}] = \tag{D.13}$$

$$nfl(2-\log(\beta)) - \log(\beta)nfl^2/2 + b^{H*}(\chi + \Gamma b^{H*} - \rho\sigma^2) - g[1+nfl]$$
$$+[\alpha^2[1+nfl]nfl + \rho^2 b^{H*}(b^{H*}-1) + 2\alpha\rho[1+nfl]b^{H*}]\frac{\sigma^2}{2}$$

Under Condition 3, $P[nfl^{opt}(b^{H*}), b^{H*}] = 0$ admits two solutions. The largest solution is close to zero and hence satisfies the condition for our quadratic approximation. We thus impose $nfl^{opt}(b^{H*})$ to be the largest solution.

(ii) For a given nfl, the gross foreign liabilities b^{H*} that satisfy portfolio optimality are characterized by

$$b^{H*} = b^{opt}(nfl) \tag{D.14}$$

where

$$b^{opt}(nfl) = \frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2}nfl \tag{D.15}$$

(iii) Under Condition 3, $P^{opt}(nfl) = 0$ admits two solutions, where $P^{opt}(nfl) = P(nfl, b^{opt}(nfl))$. Let nfl^{opt} be the largest solution to $P^{opt}(nfl) = 0$. Define b^{opt} as $b^{opt}(nfl^{opt})$. Then (nfl^{opt}, b^{opt}) jointly satisfy intertemporal and portfolio optimality.

Point (i):

Following Definition 1, intertemporal optimality is equivalent to $\lambda^F = 0$. According to Equation (D.10), $\lambda^F = 0$ implies that nfl satisfies (D.12), where $nfl^{opt}(b^{H*})$ is characterized by $P[nfl^{opt}(b^{H*}), b^{H*}] = 0$, with $P(nfl, b^{H*})$ defined by (D.13).

If $g = \Gamma = \chi = \sigma^2 = 0$, then $P(nfl, b^{H*})$ boils down to

$$P(nfl, b^{H*}) \simeq nfl[(2 - \log(\beta)) - \log(\beta) nfl/2]$$

In that case, there exists two solutions $nfl^{opt}(b^{H*})$ to $P[nfl^{opt}(b^{H*}), b^{H*}] = 0$. The existence of these two solutions extends by continuity to the case where Condition 3 is satisfied, that is, g, Γ, χ and σ^2 are small.

The largest solution corresponds to $nfl^{opt}(b^{H*}) = 0$ when $g = \Gamma = \chi = \sigma^2 = 0$, while the smallest solution is $2(2 - \log(\beta))/\log(\beta) < 0$. Only the largest solution thus satisfies the condition for our quadratic approximation to hold. Note also that it is the only well-behaved solution, as it achieves consumption smoothing. In what follows, we thus refer to $nfl^{opt}(b^{H*})$ as the largest solution to $P[nfl^{opt}(b^{H*}), b^{H*}] = 0$.

Point (ii):

Following Definition 1, portfolio optimality is equivalent to $\lambda^F = \lambda^H$. According to (D.11), $\lambda^F = \lambda^H$ implies that b^{H*} satisfies (D.14), where $b^{opt}(nfl)$ is characterized by (D.15).

Point (iii):

Suppose that there exists a couple (nfl, b^{H*}) that jointly satisfies (D.12) and (D.14). We denote this couple (nfl^{opt}, b^{opt}). nfl^{opt} is thus the value of nfl that holds under both intertemporal and portfolio optimality. It is characterized by $P(nfl^{opt}, b^{opt}(nfl^{opt})) = 0$. $P(nfl^{opt}, b^{opt}(nfl^{opt}))$ is a second-order polynomial in nfl^{opt}. We denote it by P^{opt}.

$$\begin{aligned} P^{opt}(nfl) &= nfl(2 - \log(\beta)) - \log(\beta) nfl^2/2 \\ &+ \left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right)\left[\chi + \Gamma\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right) - \rho\sigma^2\right] - g(1 + nfl) \\ &+ \left\{\alpha^2(1 + nfl) nfl + \rho^2\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right)\left[\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right) - 1\right]\right. \\ &\left. + 2\alpha\rho(1 + nfl)\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2} nfl\right)\right\}\frac{\sigma^2}{2} \end{aligned} \tag{D.16}$$

To show that solutions exist, we follow similar steps as for point (i). If $g = \Gamma =$

$\chi = \sigma^2 = 0$, then this equation boils down to

$$P^{opt}(nfl) \simeq nfl[(2 - \log(\beta)) - \log(\beta)nfl/2]$$

In that case, there exists two solutions to $P^{opt}(x) = 0$. The existence of these two solutions extends by continuity to the case where Condition 3 is satisfied, that is, g, Γ, χ and σ^2 are small. In what follows, we refer to nfl^{opt} as the largest solution to $P^{opt}(x) = 0$.

We then derive b^{opt} as $b^{opt}(nfl^{opt})$, which is uniquely defined.

D.4.2 Properties of (nfl, b^{H*})

It is useful to define b^{max} as follows:

$$b^{max} = b^{opt}(\min\{nfl^{opt}, b^G\}) = \frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2}\min\{nfl^{opt}, b^G\} \tag{D.17}$$

The following lemma characterizes some properties of the equilibrium solution for (nfl, b^{H*}).

Lemma 3 (Properties) *If $b^{CBF} + b^G \leq b^{max}$, then*

(i) $b^{H*} \leq b^{max}$*, with a strict inequality if* $b^{CBF} + b^G < b^{max}$.

If, additionally, Conditions 1, 2 and 3 are satisfied,

(ii) $nfl^{opt}(b^{H*})$ *is strictly increasing in* b^{H*} *and* $b^{opt}(nfl)$ *is decreasing in* nfl*. If, additionally,* $\alpha = 0$*, then* $b^{opt}(nfl)$ *is invariant in* nfl $\alpha > 0$*. If, instead,* $\alpha > 0$*, then* $b^{opt}(nfl)$ *is* strictly *decreasing in* nfl.

(iii) Then, $nfl \leq nfl^{opt}(b^{H*})$*, with*

$$nfl < nfl^{opt}(b^{H*}) \Leftrightarrow \lambda^F > 0$$

(iv) $b^{H*} \leq b^{opt}(nfl)$ *and* $\lambda^H - \lambda^F \geq 0$*, with strict inequalities if* $b^{CBF} + b^G < b^{max}$.

(v) The net foreign liabilities nfl *satisfy:*

$$nfl = \min\{nfl^{opt}(b^{H*}), b^G\} \tag{D.18}$$

Point (i):

Because of the household's domestic bond short-selling constraint, $b^{H} = b^{CBF} + b^{G} - b^{H*} \geq 0$. Therefore, $b^{H*} \leq b^{CBF} + b^{G}$. Since, by assumption, $b^{CBF} + b^{G} \leq b^{max}$, then $b^{H*} \leq b^{max}$. If $b^{CBF} + b^{G} < b^{max}$, then $b^{H*} < b^{max}$.

Point (ii):

According to Equation (D.15), it is straightforward that, if $\alpha \geq 0$ and $\rho > 0$, which is the case under Condition 1, then $b^{opt\prime}(nfl) \leq 0$. If, additionally, $\alpha = 0$, then $b^{opt}(nfl) = \frac{\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2}$ and $b^{opt\prime}(nfl) = 0$. If instead $\alpha > 0$, then $b^{opt\prime}(nfl) > 0$.

We can also show that $nfl^{opt\prime}(b^{H*}) > 0$ under some sufficient condition, by differentiating $P[nfl^{opt}(b^{H*}), b^{H*}]$ with respect to b^{H*}:

$$nfl^{opt\prime}(b^{H*}) P_1[nfl^{opt}(b^{H*}), b^{H*}] + P_2[nfl^{opt}(b^{H*}), b^{H*}] = 0$$

$$nfl^{opt\prime}(b^{H*}) \underbrace{P_1[nfl^{opt}(b^{H*}), b^{H*}]}_{>0} + \underbrace{\chi + (2\Gamma + \rho^2\sigma^2) b^{H*} - \rho\sigma^2 + \alpha\rho[1 + nfl^{opt}(b^{H*})]\sigma^2 - \rho^2\sigma^2/2}_{<0 \text{ under Conditions 1, 2, and 3, and if } b^{H*} \leq b^{max}} = 0$$

$P_1[nfl^{opt}(b^{H*}), b^{H*}] > 0$ because $nfl^{opt}(b^{H*})$ is the largest solution to $P[nfl^{opt}(b^{H*}), b^{H*}] = 0$ and P opens upwards.

According to point (i), we have $b^{H*} \leq b^{max}$, because by assumption $b^{CBF} + b^{G} \leq b^{max}$. We show in what follows that under Conditions 1, 2 and 3, and if $b^{H*} \leq b^{max}$, the second line in the above equation is strictly negative. Indeed, if $b^{H*} \leq b^{max}$, then:

$$\begin{aligned} & \chi + (2\Gamma + \rho^2\sigma^2) b^{H*} - \rho\sigma^2 + \alpha\rho[1 + nfl^{opt}(b^{H*})]\sigma^2 - \rho^2\sigma^2/2 \\ \leq \; & \chi + (2\Gamma + \rho^2\sigma^2) b^{max} - \rho\sigma^2 + \alpha\rho[1 + nfl^{opt}(b^{H*})]\sigma^2 - \rho^2\sigma^2/2 \end{aligned}$$

Using the definition of b^{max} (D.17):

$$\begin{aligned} & \chi + (2\Gamma + \rho^2\sigma^2) b^{H*} - \rho\sigma^2 + \alpha\rho[1 + nfl^{opt}(b^{H*})]\sigma^2 - \rho^2\sigma^2/2 \\ \leq \; & \underbrace{\chi + (2\Gamma + \rho^2\sigma^2)\left(\frac{(1-\alpha)\rho\sigma^2 - \chi}{\Gamma + \rho^2\sigma^2} - \frac{\alpha\rho\sigma^2}{\Gamma + \rho^2\sigma^2}\min\{nfl^{opt}, b^{G}\}\right) - \rho\sigma^2 + \alpha\rho[1 + nfl^{opt}(b^{H*})]\sigma^2 - \rho^2\sigma^2/2}_{X} \end{aligned}$$

We now use the Conditions to find an approximation for X and show that X is strictly negative. Using Condition 2, we can neglect the terms in Γ and χ:

$$X \simeq -\rho\sigma^2 + [\rho(1-\alpha) + \alpha\rho[1 + nfl^{opt}(b^{H*})]]\sigma^2 - \rho^2\sigma^2/2 - \alpha\rho\sigma^2 \min\{nfl^{opt}, b^{G}\}$$

Besides, using the fact that, under Condition 3, $nfl^{opt}(b^{H*})$ is small:

$$X \simeq -\rho\sigma^2 + [\rho(1-\alpha) + \alpha\rho]\sigma^2 - \rho^2\sigma^2/2 - \frac{\alpha}{\rho}\min\{nfl^{opt}, b^{G}\} = -\rho^2\sigma^2/2 - \alpha\rho\sigma^2 \min\{nfl^{opt}, b^{G}\}$$

Finally, we consider two cases. First, suppose that $\min\{nfl^{opt}, b^G\} = b^G$. Noting that b^G is strictly positive, and using Condition 1, according to which $\alpha \geq 0$ and $\rho > 0$, we can write

$$X \simeq -\rho^2\sigma^2/2 - \alpha\rho\sigma^2 b^G < -\rho^2\sigma^2/2 < 0$$

Second, suppose that $\min\{nfl^{opt}, b^G\} = nfl^{opt}$. Using the fact that, under Condition 3, nfl^{opt} is close to zero:

$$X \simeq -\rho^2\sigma^2/2 - \alpha\rho\sigma^2 nfl^{opt} \simeq -\rho^2\sigma^2/2 < 0$$

As a result, $nfl^{opt\prime}(b^{H*})$ is strictly positive.

Point (iii):

According to Equation (D.10),

$$\lambda^F > 0 \Leftrightarrow P(nfl, b^{H*}) < 0$$

Now, note that $P(nfl^{opt}(b^{H*}), b^{H*}) = 0$. As a result,

$$\lambda^F > 0 \Leftrightarrow P(nfl, b^{H*}) < P(nfl^{opt}(b^{H*}), b^{H*})$$

Taking also into account the fact that $nfl^{opt}(b^{H*})$ is the largest solution to $P[nfl^{opt}(b^{H*}), b^{H*}] = 0$ and that P opens upwards, then we can infer

$$\lambda^F > 0 \Leftrightarrow nfl < nfl^{opt}(b^{H*})$$

Finally, taking into account the fact that $\lambda^F = 0 \Leftrightarrow nfl = nfl^{opt}(b^{H*})$ (by definition of $nfl^{opt}(b^{H*})$), the above equivalence implies that $\lambda^F \geq 0 \Leftrightarrow nfl \leq nfl^{opt}(b^{H*})$. As a result, since $\lambda^F \geq 0$, then necessarily $nfl \leq nfl^{opt}(b^{H*})$.

Point (iv):

Remember that $b^{max} = b^{opt}[\min\{nfl^{opt}, b^G\}]$. Either $b^{max} = b^{opt}(nfl^{opt}) = b^{opt}$, or $b^{max} = b^{opt}(b^G)$. We consider these two cases in turn, and show that they lead to $b^{H^*} \leq b^{opt}(nfl)$, with a strict inequality if $b^{CBF} + b^G < b^{max}$.

- First, consider the case $b^{max} = b^{opt}$. According to Lemma 3, point (i), $b^{CBF} + b^G \leq b^{max}$ implies that $b^{H*} \leq b^{max}$ (with a strict inequality when $b^{CBF} + b^G < b^{max}$). As a consequence, $b^{H*} \leq b^{opt}$, and hence $nfl^{opt}(b^{H*}) \leq nfl^{opt}(b^{opt}) = nfl^{opt}$. Indeed, $nfl^{opt}(.)$ is an increasing function, according to Lemma 3, point (ii). As a consequence, $\min\{nfl^{opt}(b^{H*}), b^G\} \leq \min\{nfl^{opt}, b^G\}$. This implies $b^{opt}(\min\{nfl^{opt}(b^{H*}), b^G\}) \geq b^{opt}(\min\{nfl^{opt}, b^G\}) = b^{max}$, because $b^{opt}(.)$ is a decreasing function, according to Lemma 3, point (ii).

On the other hand, $b^{opt}(nfl) \geq b^{opt}(\min\{nfl^{opt}(b^{H*}), b^G\})$. Indeed, note that $nfl = b^G - b^H - b^F$, so the short-selling constraints imply $nfl \leq b^G$. Moreover, according to Lemma 3, point (iii), $nfl \leq nfl^{opt}(b^{H*})$. As a consequence, $nfl \leq \min\{nfl^{opt}(b^{H*}), b^G\}$. Therefore, $b^{opt}(nfl) \geq b^{opt}(\min\{nfl^{opt}(b^{H*}), b^G\})$.

Combining this with $b^{opt}(\min\{nfl^{opt}(b^{H*}), b^G\}) \geq b^{max}$, we obtain $b^{opt}(nfl) \geq b^{max}$. Since $b^{H*} \leq b^{max}$ (with a strict inequality when $b^{CBF} + b^G < b^{max}$), then it implies $b^{opt}(nfl) \geq b^{H*}$ (with a strict inequality when $b^{CBF} + b^G < b^{max}$).

- Second, consider the case $b^{max} = b^{opt}(b^G)$. Then, because $nfl \leq b^G$, $b^{opt}(nfl) \geq b^{opt}(b^G) = b^{max}$, since $b^{opt}(.)$ is a decreasing function, according to Lemma 3, point (ii). Since $b^{H*} \leq b^{max}$ (with a strict inequality when $b^{CBF} + b^G < b^{max}$), then it implies $b^{opt}(nfl) \geq b^{H*}$ (with a strict inequality when $b^{CBF} + b^G < b^{max}$).

We have thus shown that $b^{H^*} \leq b^{opt}(nfl)$, with a strict inequality when $b^{CBF} + b^G < b^{max}$.

On the other hand, according to Equation (D.11),

$$\lambda^H \geq \lambda^F \Leftrightarrow (\Gamma + \rho^2\sigma^2)b^{H^*} - (1-\alpha)\rho\sigma^2 + \chi + \alpha\rho\sigma^2 nfl \leq 0$$

Now, note that, by definition of $b^{opt}(.)$, $(\Gamma + \rho^2\sigma^2)b^{opt}(nfl) - (1-\alpha)\rho\sigma^2 + \chi + \alpha\rho\sigma^2 nfl = 0$. As a result,

$$\begin{aligned}
&\lambda^H \geq \lambda^F \\
&\Leftrightarrow (\Gamma + \rho^2\sigma^2)b^{H^*} - (1-\alpha)\rho\sigma^2 + \chi + \alpha\rho\sigma^2 nfl \leq (\Gamma + \rho^2\sigma^2)b^{opt}(nfl) - (1-\alpha)\rho\sigma^2 + \chi + \alpha\rho\sigma^2 nfl \\
&\Leftrightarrow b^{H^*} \leq b^{opt}(nfl)
\end{aligned}$$

Since $b^{H^*} \leq b^{opt}(nfl)$, this means that, necessarily, $\lambda^H \geq \lambda^F$. Because the former inequality holds strictly when $b^{CBF} + b^G < b^{max}$, we can show that, in that case, $\lambda^H \lambda^F$.

Point (v):

First, we show that $nfl \leq \min\{nfl^{opt}(b^{H*}), b^G\}$. Indeed, note that $nfl = b^G - b^H - b^F$, so the short-selling constraints imply $nfl \leq b^G$. Moreover, according to Lemma 3 point (iii), $nfl \leq nfl^{opt}(b^{H*})$. As a consequence, $nfl \leq \min\{nfl^{opt}(b^{H*}), b^G\}$.

As a result, either $nfl < \min\{nfl^{opt}(b^{H*}), b^G\}$ or $nfl = \min\{nfl^{opt}(b^{H*}), b^G\}$. We now assume that $nfl < \min\{nfl^{opt}(b^{H*}), b^G\}$ and show that this leads to a contradiction.

Note that this assumption implies that $nfl < nfl^{opt}(b^{H*})$. According to Lemma 3, point (iii), this means that $\lambda^F > 0$ and $b^F = 0$. It also implies that $nfl < b^G$, which means that $b^G - nfl = b^H > 0$, and hence $\lambda^H = 0$.

Since $\lambda^F > 0$ and $\lambda^H = 0$, then $\lambda^H - \lambda^F < 0$. This would contradict Lemma 3 point (iv).

Therefore, $nfl < \min\{nfl^{opt}(b^{H*}), b^G\}$ does not hold. The only possible solution is $nfl = \min\{nfl^{opt}(b^{H*}), b^G\}$.

D.4.3 b^{max} and portfolio optimality

Lemma 4 **(b^{max})** *Under Conditions 1, 2 and 3, and if* $b^{CBF} + b^G = b^{max}$*, then* $\lambda^H - \lambda^F = 0$*, and portfolio optimality is satisfied.*

According to Lemma 3, point (iv), $\lambda^H - \lambda^F \geq 0$ and, equivalently, $b^{H*} \leq b^{opt}(nfl)$. We now suppose that $\lambda^H - \lambda^F > 0$, and, equivalently, $b^{H*} < b^{opt}(nfl)$ and show that it leads to contradiction.

If $\lambda^H - \lambda^F > 0$, then, because $\lambda^F \geq 0$, necessarily $\lambda^H > 0$. This implies that $b^H = 0$. As a consequence, $b^{H^*} = b^{CBF} + b^G$. Because $b^{CBF} + b^G = b^{max}$, this implies that $b^{H^*} = b^{max}$.

Since $\lambda^H - \lambda^F > 0$ is equivalent to $b^{H*} < b^{opt}(nfl)$, then this also implies $b^{max} < b^{opt}(nfl)$. By definition, $b^{max} = b^{opt}(\min\{b^G, nfl^{opt}\})$. Moreover, according to Lemma 3, point (v), $nfl = \min\{b^G nfl^{opt}(b^{H*})\}$. As a consequence, $b^{max} < b^{opt}(nfl)$ is equivalent to

$$b^{opt}(\min\{b^G, nfl^{opt}\}) < b^{opt}(\min\{b^G nfl^{opt}(b^{H*})\}) \tag{D.19}$$

According to Lemma 3, point (ii), $b^{opt}(.)$ is constant if $\alpha = 0$. In that case, inequality (D.19) cannot hold and we have a contradiction. If $\alpha > 0$, then $b^{opt}(.)$ is a strictly decreasing function, so that we have

$$\min\{b^G, nfl^{opt}\} > \min\{b^G, nfl^{opt}(b^{H*})\} \tag{D.20}$$

We next consider 4 cases:

- $b^G < nfl^{opt}$ and $b^G < nfl^{opt}(b^{H*})$. Then (D.20) implies $b^G > b^G$, which leads to a contradiction.

- $b^G > nfl^{opt}$ and $b^G > nfl^{opt}(b^{H*})$. Then (D.20) implies $nfl^{opt} > nfl^{opt}(b^{H*})$. On the other hand, $b^{H*} = b^{opt}(\min\{b^G, nfl^{opt}\})$ implies $b^{H*} = b^{opt}(nfl^{opt}) = b^{opt}$ and $nfl^{opt}(b^{H^*}) = nfl^{opt}(b^{opt}) = nfl^{opt}$, so that the inequality implies $nfl^{opt} > nfl^{opt}$, which leads to a contradiction.

- $b^G > nfl^{opt}$ and $b^G < nfl^{opt}(b^{H*})$. Then (D.20) implies $nfl^{opt} > b^G$, which leads to a contradiction.

- $b^G < nfl^{opt}$ and $b^G > nfl^{opt}(b^{H*})$. This implies that $nfl^{opt}(b^{H*}) < nfl^{opt}$. Moreover, $b^{H*} = b^{opt}(\min\{b^G, nfl^{opt}\})$ implies $b^{H*} = b^{opt}(b^G)$ and hence $nfl^{opt}(b^{H*}) < nfl^{opt}$ implies $nfl^{opt}(b^{opt}(b^G)) < nfl^{opt}$. Now, note that $nfl^{opt} = nfl^{opt}(b^{opt}(nfl^{opt}))$. Therefore, $nfl^{opt}(b^{opt}(b^G)) < nfl^{opt}(b^{opt}(nfl^{opt}))$. Since, according to Lemma 3, point (ii), $nfl^{opt}(.)$ is strictly increasing, while $b^{opt}(.)$ is strictly decreasing, this inequality implies that $b^G > nfl^{opt}$, which leads to a contradiction.

Therefore, neither inequality (D.20), nor (D.19), can hold, so the case where $\lambda^H - \lambda^F > 0$ can be ruled out. By contradiction, we must have $\lambda^H - \lambda^F = 0$. According to Equation (14) and the definition of portfolio optimality 1, portfolio optimality is thus satisfied.

D.4.4 Effectiveness of FX interventions

Lemma 3, point (iv), states that, if $b^{CBF} + b^G < b^{max}$, and if Conditions 1, 2 and 3 are satisfied, then $\lambda^H - \lambda^F > 0$. Because $\lambda^F \geq 0$, this implies that $\lambda^H > 0$. As a result, $b^H = 0$. Additionally, because $b^H = b^{CBF} + b^G - b^{H*}$, then $b^{H*} = b^{CBF} + b^G$. This proves the first part of Proposition 1.

The impact of FX interventions on the CIP deviation Z^* and on the UIP deviation $\mathbb{E}(X^*)$ derive immediately from Equations (8) and (7) and the effect of interventions on b^{H*}. This proves the second part of Proposition 1.

D.5 The Impact of FX Interventions on nfl

Lemma 5 *If Conditions 1, 2 and 3 are satisfied, and $b^{CBF} + b^G < b^{max}$, then the following holds:*

(i) If b^G is small, so that $b^G < nfl^{opt}(b^{CBF} + b^G)$, then $nfl = b^G$ and the households are constrained not only in their capacity to issue domestic-currency

bonds ($\lambda^H > 0$), but also in their capacity to issue foreign-currency bonds ($\lambda^F > 0$). In that case, nfl is invariant in b^{CBF}.

(ii) If b^G is large, so that $b^G \geq nfl^{opt}(b^{CBF} + b^G)$, then $nfl = nfl^{opt}(b^{CBF} + b^G)$ and the households are only constrained in their capacity to issue domestic-currency bonds ($\lambda^H > 0$ and $\lambda^F = 0$). In that case, nfl is increasing in b^{CBF}.

We assume that Conditions 1, 2 and 3 are satisfied, and that $b^{CBF} + b^G < b^{max}$.

According to Proposition 1, since $b^{CBF} + b^G < b^{max}$, then $b^{H*} = b^{CBF} + b^G$, and households are constrained in their capacity to issue domestic-currency bonds ($b^H = 0$).

According to Lemma 3, point (v), under Conditions 1, 2 and 3, and $b^{CBF} + b^G < b^{max}$, $nfl = \min\{nfl^{opt}(b^{H*}), b^G\}$. Besides, since $b^{H*} = b^{CBF} + b^G$, then $nfl = \min\{nfl^{opt}(b^{CBF} + b^G), b^G\}$.

This means that, if b^G is small, so that $b^G < nfl^{opt}(b^{CBF} + b^G)$, then $nfl = b^G$ and the households are constrained not only in their capacity to issue domestic-currency bonds, but also in their capacity to issue foreign-currency bonds ($\lambda^H > 0$ and $\lambda^F > 0$).

If b^G is large, so that $b^G \geq nfl^{opt}(b^{CBF} + b^G)$, then $nfl = nfl^{opt}(b^{CBF} + b^G)$. In that case, intertemporal optimality is satisfied as households desire a positive amount of foreign-currency bonds ($b^F = b^G - nfl \geq 0$). According to Lemma 3 point (ii), in that case, $nfl^{opt}(b^{CBF} + b^G)$ is increasing in $b^{CBF} + b^G$. Therefore, nfl is increasing in b^{CBF}.

D.6 Covariance differential

The difference in risk premia can be written as

$$\Delta Cov = \frac{cov(m^*, X^*)}{\mathbb{E}m^*} - \frac{cov(m, X^*)}{\mathbb{E}m}$$

We compute first $cov(m^*, X^*)/\mathbb{E}m^*$:

$$\begin{aligned}
\frac{cov(m^*, X^*)}{\mathbb{E}(m^*)} &= \frac{cov\left(m^*, (1+i)\frac{S_1}{S_2}\right)}{\mathbb{E}(m^*)} - \frac{\overbrace{cov(m^*, (1+i^*))}}{\underset{=0}{}\mathbb{E}(m^*)} = \frac{\mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right)}{\mathbb{E}(m^*)} - \mathbb{E}\left((1+i)\frac{S_1}{S_2}\right) \\
&= \frac{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}(m^*)} - \mathbb{E}\left(e^{\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right) = \frac{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}(m^*)}\left(1 - \frac{\mathbb{E}(e^{\tilde{m}^*})\mathbb{E}\left(e^{\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}\right) \\
&= \frac{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}(m^*)}\left(1 - \frac{\mathbb{E}(e^{\tilde{m}^*})\mathbb{E}\left(e^{-\tilde{S}_2}\right)}{\mathbb{E}\left(e^{\tilde{m}^*-\tilde{S}_2}\right)}\right) \\
&= \frac{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}(m^*)}\left(1 - \frac{e^{\mathbb{E}(\tilde{m}^*)+V(\tilde{m}^*)/2-\mathbb{E}(\tilde{S}_2)+V(\tilde{S}_2)/2}}{e^{\mathbb{E}(\tilde{m}^*-\tilde{S}_2)+V(\tilde{m}^*-\tilde{S}_2)/2}}\right) \\
&= \underbrace{\frac{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}(m^*)}}_{\frac{1}{\beta}(1+\chi+\Gamma b^{H*})}\left[1 - e^{cov(\tilde{S}_2,\tilde{m}^*)}\right] \qquad \text{(D.21)}
\end{aligned}$$

where we used (D.2), and the assumption that $\mathbb{E}m^* = \beta$.

Similarly:

$$\begin{aligned}
\frac{cov(m, X^*)}{\mathbb{E}m} &= \frac{cov\left(m, (1+i)\frac{S_1}{S_2}\right) - \underbrace{cov(m, (1+i^*))}_{=0}}{\mathbb{E}m} = \frac{\mathbb{E}\left(m(1+i)\frac{S_1}{S_2}\right)}{\mathbb{E}m} - \mathbb{E}\left((1+i)\frac{S_1}{S_2}\right) \\
&= \frac{\mathbb{E}\left(e^{\tilde{m}+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}\left(e^{\tilde{m}}\right)} - \mathbb{E}\left(e^{\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right) = \frac{\mathbb{E}\left(e^{\tilde{m}+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}\left(e^{\tilde{m}}\right)}\left(1 - \frac{\mathbb{E}\left(e^{\tilde{m}}\right)\mathbb{E}\left(e^{\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}{\mathbb{E}\left(e^{\tilde{m}+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}\right) \\
&= e^{\tilde{i}+\tilde{S}_1+\mathbb{E}(\tilde{m}-\tilde{S}_2)+V(\tilde{m}-\tilde{S}_2)/2-\mathbb{E}(\tilde{m})-V(\tilde{m})/2}\left(1 - \frac{e^{\tilde{i}+\tilde{S}_1+\mathbb{E}(\tilde{m}-\tilde{S}_2)+V(\tilde{m})/2+V(\tilde{S}_2)/2}}{e^{\tilde{i}+\tilde{S}_1+\mathbb{E}(\tilde{m}-\tilde{S}_2)+V(\tilde{m}-\tilde{S}_2)/2}}\right) \\
&= e^{\tilde{i}+\tilde{S}_1-\mathbb{E}(\tilde{S}_2)+V(\tilde{S}_2)/2-cov(\tilde{S}_2,\tilde{m})}\left(1 - e^{cov(\tilde{S}_2,\tilde{m})}\right) \\
&= \frac{1}{\beta}e^{\tilde{i}+\tilde{S}_1-\mathbb{E}(\tilde{S}_2)+V(\tilde{S}_2)/2+\mathbb{E}(\tilde{m}^*)+\frac{V(\tilde{m}^*)}{2}-cov(\tilde{S}_2,\tilde{m}^*)}\left[e^{cov(\tilde{S}_2,\tilde{m}^*)-cov(\tilde{S}_2,\tilde{m})} - e^{cov(\tilde{S}_2,\tilde{m}^*)}\right] \\
&= \frac{1}{\beta}\underbrace{\mathbb{E}\left(e^{\tilde{m}^*+\tilde{i}+\tilde{S}_1-\tilde{S}_2}\right)}_{1+\chi+\Gamma b^{H*}}\left[e^{cov(\tilde{S}_2,\tilde{m}^*)-cov(\tilde{S}_2,\tilde{m})} - e^{cov(\tilde{S}_2,\tilde{m}^*)}\right] \qquad \text{(D.22)}
\end{aligned}$$

where we used (D.2) and $\beta = \mathbb{E}(m^*) = \mathbb{E}(e^{\tilde{m}^*}) = e^{\mathbb{E}(\tilde{m}^*)+V(\tilde{m}^*)/2}$.

This yields

$$\begin{aligned}\Delta Cov &= \frac{cov(m^*, X^*)}{\mathbb{E}m^*} - \frac{cov(m, X^*)}{\mathbb{E}m} \\ &= \frac{1}{\beta}(1+\chi+\Gamma b^{H*})\left[1 - e^{cov(\tilde{S}_2,\tilde{m}^*)-cov(\tilde{S}_2,\tilde{m})}\right] \qquad \text{(D.23)}\end{aligned}$$

Now, remember that, by assumption, $\tilde{m}^* = \log(\beta) - \tilde{y}^*$, $\tilde{S}_2 = \rho[\tilde{y}^* - (1-\rho)\sigma^2/2]$, and consider Equation (D.5). This implies

$$cov(\tilde{S}_2, \tilde{m}^*) = -\rho\sigma^2$$

$$cov(\tilde{S}_2, \tilde{m}) = -\rho[\alpha(1+nfl) + \rho b^{H*}]\sigma^2$$

Therefore,

$$\Delta Cov = \frac{1}{\beta}(1+\chi+\Gamma b^{H*})\left[1 - e^{-\rho\sigma^2[1-\alpha(1+nfl)-\rho b^{H*}]}\right]$$

A linear approximation of this equation yield Equation (28).

D.7 Proof of Proposition 3

According to Equation (27), $UCFX = -(\lambda^H - \lambda^F)/\mathbb{E}(m)$. Now, note that, according to (13), $\mathbb{E}(m) = (1-\lambda^F)/(1+i^*) = \beta(1-\lambda^F)$, so that $UCFX = -(\lambda^H - \lambda^F)/\beta(1-\lambda^F)$.

On the other hand, note that, according to Equation (D.11), $(\lambda^H - \lambda^F)/(1-\lambda^F)$ depend negatively on b^{H*} and nfl. Since, according to Lemma 5, point (i), $nfl = \min\{nfl^{opt}(b^{H*}), b^G\}$, and since, according to Lemma 3, point (ii), $nfl^{opt}(b^{H*})$ is increasing in b^{H*}, while b^G is invariant in b^{H*}, then, nfl is itself increasing in b^{H*}. As a consequence, $(\lambda^H - \lambda^F)/(1-\lambda^F)$ is decreasing in b^{H*}. Therefore, $UCFX$ is increasing in b^{H*}.

Additionally, since Conditions 1, 2 and 3 hold, and $b^{CBF} + b^G < b^{max}$, Proposition 1 holds. Therefore, $b^{H*} = b^{CBF} + b^G$ and b^{H*} is increasing in b^{CBF}. As a result, $UCFX$ is increasing in b^{CBF}.

According to Lemma 3, point (iv), under Conditions 1, 2 and 3, and $b^{CBF} + b^G < b^{max}$, we have $\lambda^H - \lambda^F > 0$. As a result, $UCFX = -(\lambda^H - \lambda^F)/\mathbb{E}(m) < 0$.

E Proofs - Constrained Planner Program

In this section, we derive some key properties of the constrained planner equilibrium. In E.1, we derive the other FOCs of the constrained planner and the planner's stochastic discount factor. In E.2, we derive the planner's optimality condition (37). E.3 proves Lemma 1 and E.4 proves Proposition 5 .

E.1 Other FOCs and the Planner's Stochastic Discount Factor

We take the first-order conditions of the constrained planner's program with respect to prices:

$$/i: \quad -\mathbb{E}\left[\eta_2(1+i)\frac{S_1}{S_2}b^{H*}\right] + \xi f_1(i,S_1)(1+i) \quad +\alpha_0\mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right) = 0 \tag{E.1}$$

$$/S_1: \quad -\mathbb{E}\left[\eta_2(1+i)\frac{S_1}{S_2}b^{H*}\right] + \xi f_2(i,S_1)S_1 \quad +\alpha_0\mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right) = 0 \tag{E.2}$$

These two equations imply that $\xi f_1(i,S_1)(1+i) = \xi f_2(i,S_1)S_1$. This is true in the general case only if $\xi = 0$ (you can see that by setting $f(i,S_1) = S_1 - 1$ for instance). The shadow cost of the "policy constraint" is zero, because of monetary neutrality.

This leads to

$$\mathbb{E}\left[\eta_2(1+i)\frac{S_1}{S_2}b^{H*}\right] = \alpha_0\mathbb{E}\left(m^*(1+i)\frac{S_1}{S_2}\right) \tag{E.3}$$

Finally, we derive with respect to consumption:

$$/C_1: \qquad U'(C_1) - \eta_1 = 0 \tag{E.4}$$

$$/C_2: \qquad \mathbb{E}\left[\beta U'(C_2) - \eta_2\right] = 0 \tag{E.5}$$

These equations imply that $m^{CB} = \eta_2/\eta_1 = \beta U'(C_2)/U'(C_1) = m$.

E.2 Optimal foreign exchange interventions

Equation (E.3) yields

$$\frac{\alpha_0}{\eta_1} = b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} \tag{E.6}$$

where we have used $\mathbb{E}(\eta_2/\eta_1) = m$. α_0 is of the same sign as b^{H*}, the gross external position in domestic currency. In that case, if the country is short in domestic currency, then α_0 is positive.

Dividing Equation (35) by η_1, replacing η_2/η_1 with m and α_0/η_1 using the above expression, and using $\Lambda - \tilde{\Lambda} = 0$ (Equation (36)), then finally dividing by $\mathbb{E}m$, we obtain

$$-\frac{\mathbb{E}(mX^*)}{\mathbb{E}m} - \Gamma b^{H*} \frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}m\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} = 0$$

This yields equation (37).

E.3 Proof of Lemma 1

Point (i)

Another way to write Equation (37) is:

$$\begin{aligned}
\mathbb{E}\left(m(1+i^*)\right) - \mathbb{E}\left(m(1+i)\frac{S_1}{S_2 2}\right) - \Gamma b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} &= 0 \\
\underbrace{(1+i^*)}_{\frac{1}{\mathbb{E}(m^*)}}\mathbb{E}(m) - \underbrace{(1+i)}_{\frac{1+\chi+\Gamma b^{H*}}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}}\mathbb{E}\left(m\frac{S_1}{S_2}\right) - \Gamma b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} &= 0 \\
\frac{1}{\mathbb{E}(m^*)} - \frac{1+\chi+\Gamma b^{H*}}{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}(m)} - \Gamma b^{H*}\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}(m)\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)} &= 0 \\
1 - (1+\chi+\Gamma b^{H*})\frac{\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}(m)}}{\frac{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}{\mathbb{E}(m^*)}} - \Gamma b^{H*}\frac{\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}(m)}}{\frac{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}{\mathbb{E}(m^*)}} &= 0 \\
1 - (1+\chi+2\Gamma b^{H*})\frac{\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}(m)}}{\frac{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}{\mathbb{E}(m^*)}} &= 0
\end{aligned}$$

where we used (D.2). Besides,

$$\frac{\frac{\mathbb{E}\left(m\frac{S_1}{S_2}\right)}{\mathbb{E}(m)}}{\frac{\mathbb{E}\left(m^*\frac{S_1}{S_2}\right)}{\mathbb{E}(m^*)}} = \frac{\frac{e^{\tilde{S}_1+\mathbb{E}(\tilde{m}-\tilde{S}_2)+V(\tilde{m}-\tilde{S}_2)/2}}{e^{\mathbb{E}(\tilde{m})+V(\tilde{m})/2}}}{\frac{e^{\tilde{S}_1+\mathbb{E}(\tilde{m}^*-\tilde{S}_2)+V(\tilde{m}^*-\tilde{S}_2)/2}}{e^{\mathbb{E}(\tilde{m}^*)+V(\tilde{m}^*)/2}}} = e^{cov(\tilde{m}^*,\tilde{S}_2)-cov(\tilde{m},\tilde{S}_2)} = e^{-\rho\sigma^2[1-\alpha(1+nfl)-\rho b^{H*}]} \tag{E.7}$$

Therefore, Equation (37) can be written as

$$e^{\rho\sigma^2[1-\alpha(1+nfl)-\rho b^{H*}]} = (1+\chi+2\Gamma b^{H*})$$

which can be approximated as

$$\rho\sigma^2[1-\alpha(1+nfl)-\rho b^{H*}] = \chi + 2\Gamma b^{H*}$$

This yields an optimal level of gross foreign liabilities $\widehat{b}(nfl)$:

$$\widehat{b}(nfl) = \frac{(1-\alpha)\rho\sigma^2-\chi}{\rho^2\sigma^2+2\Gamma} - \frac{\alpha\rho\sigma^2 nfl}{\rho^2\sigma^2+2\Gamma} \tag{E.8}$$

This optimal solution is conditional on nfl. nfl itself satisfies $nfl(\widehat{b}) = \min\{b^G, nfl^{opt}(\widehat{b})\}$, according to Lemma 5, point (i). $nfl^{opt}(b)$ is the level of net foreign liabilities that satisfies intertemporal optimality for a given b. It is the largest solution to $P(nfl^{opt}(b), b) = 0$, where P is described in (D.13) (see Lemma 2).

Suppose that there exists a couple (nfl, b) that jointly satisfies $b = \widehat{b}(nfl)$, where $\widehat{b}(nfl)$ is given by (E.8), and $P(nfl, b) = 0$. We denote this couple $(\widehat{nfl}^{opt}, \widehat{b}^{opt})$. $\widehat{nfl}^{opt}$ is thus the value of nfl that holds under both intertemporal optimality and $MBFX = 0$. It is characterized by $P(\widehat{nfl}^{opt}, \widehat{b}(\widehat{nfl}^{opt})) = 0$. $P(\widehat{nfl}^{opt}, \widehat{b}(\widehat{nfl}^{opt}))$ is a second-order polynomial in $\widehat{nfl}^{opt}$. We denote it by $\widehat{P}^{opt}$.

Following similar arguments as in Appendix D.4, we can show that under Conditions 1, 2 and 3, two solutions to $\widehat{P}^{opt}(nfl) = 0$ exist. We define the solution $\widehat{nfl}^{opt}$ as the largest of the two polynomial solutions, as argued above. Then $\widehat{b}^{opt}$ is simply defined as $\widehat{b}^{opt} = \widehat{b}(\widehat{nfl}^{opt})$.

Denote by $(\widehat{nfl}, \widehat{b})$ the solution under the constrained planner. In the constrained planner equilibrium, $MBFX = 0$, so that $\widehat{b} = \widehat{b}(\widehat{nfl})$. On the other hand, $\widehat{nfl} = \min\{b^G, nfl^{opt}(\widehat{b})\}$. If $b^G \leq nfl^{opt}(\widehat{b})$, then $\widehat{nfl} = b^G$ and $\widehat{b} = \widehat{b}(b^G)$. If $nfl^{opt}(\widehat{b}) < b^G$, then $\widehat{nfl} = nfl^{opt}(\widehat{b})$ and $\widehat{b} = \widehat{b}(\widehat{nfl})$. In that case, according to the above analysis, $\widehat{nfl} = \widehat{nfl}^{opt}$ and $\widehat{b} = \widehat{b}(\widehat{nfl}^{opt})$. Combining these two cases, $\widehat{b} = \widehat{b}(\min\{b^G, \widehat{nfl}^{opt}\})$. This yields point (i) of Lemma 1.

Point (ii)

To prove point (ii), note that, by definition of b^{max} and $\widehat{b}$, we have

$$(\rho^2\sigma^2+\Gamma)b^{max} + \alpha\rho\sigma^2 nfl(b^{max}) = (1-\alpha)\rho\sigma^2 - \chi$$

$$(\rho^2\sigma^2+2\Gamma)\widehat{b} + \alpha\rho\sigma^2 nfl(\widehat{b}) = (1-\alpha)\rho\sigma^2 - \chi$$

with $nfl(b) = \min\{b^G, nfl^{opt}(b)\}$.

Since $b^{max} > b^G$, and $b^G \geq 0$, then $b^{max} > 0$, which implies that

$$(\rho^2\sigma^2+2\Gamma)b^{max}+\alpha\rho\sigma^2 nfl(b^{max}) \geq (\rho^2\sigma^2+\Gamma)b^{max}+\alpha\rho\sigma^2 nfl(b^{max}) = (\rho^2\sigma^2+2\Gamma)\widehat{b}+\alpha\rho\sigma^2 nfl(\widehat{b})$$

if $\Gamma \geq 0$, with a strict inequality if $\Gamma > 0$. Under Conditions 1, 2 and 3, $nfl(b)$ is weakly increasing in b for $b \leq b^{max}$ (Lemma 3). We can also show that, under the same conditions, $nfl(b)$ is weakly increasing in b for $b \leq \widehat{b}$ (using similar arguments as for Lemma 3). Therefore, $nfl(b)$ is weakly increasing in b for $b \leq \max\{b^{max}, \widehat{b}\}$. As a result, $(\rho^2\sigma^2 + 2\Gamma)b + \alpha\rho\sigma^2 nfl(b)$ is strictly increasing in b for $b \leq \max\{b^{max}, \widehat{b}\}$. Therefore, $\widehat{b} \leq b^{max}$. If $\Gamma > 0$, this inequality is strict.

Point (iii)

Suppose that $\Gamma > 0$. According to (37), this implies that $UCFX < 0$. As a consequence, according to Equation (27), $\lambda^H - \lambda^F > 0$, which then implies that $\lambda^H > 0$, and hence $b^H = 0$. Since $\widehat{b} = b^{CBF} + b^G - b^H$, this implies that $\widehat{b} = b^{CBF} + b^G$, hence the result.

If $\Gamma = 0$, then, according to (37), $UCFX = 0$. According to (27), this implies that $\lambda^H - \lambda^F = 0$. We then distinguish two cases:

- Consider first the case where $b^G < \widehat{nfl^{opt}}$. Noting that, by definition, $\widehat{nfl^{opt}} = nfl^{opt}(\widehat{b})$, this means that $b^G < nfl^{opt}(\widehat{b})$. Then, according to Lemma 5, point (i), $\lambda^F > 0$, and in that case, $\lambda^H > 0$ and hence $b^H = 0$. As a consequence, $\widehat{b} = b^{CBF} + b^G$.

- Consider then the case where $b^G \geq \widehat{nfl^{opt}}$. Then, similarly, this means that $b^G \geq nfl^{opt}(\widehat{b})$. Then, according to Lemma 5, point (ii), $\lambda^F = 0$, and in that case, $\lambda^H = 0$ and hence $b^H \geq 0$. As a consequence, $\widehat{b} \leq b^{CBF} + b^G$.

E.4 Proof of Proposition 5

Note that the CIP deviation, as defined in (8), is increasing in b^{H*} (hence (i)), since $\mathbb{E}(m^*) = \beta$ is fixed. Moreover, note that the UIP deviation can be written as (we use Equations (7), (D.21) and $\mathbb{E}(m^*) = \beta$):

$$\begin{aligned}\mathbb{E}X^* &= \tfrac{1}{\beta}\left[\chi + \Gamma b^{H*} - (1 + \chi + \Gamma b^{H*})(1 - e^{-\rho\sigma^2})\right] \\ &= -\tfrac{1}{\beta}\left[1 - (1 + \chi + \Gamma b^{H*})e^{-\rho\sigma^2}\right]\end{aligned}$$

It can be approximated as follows:

$$\mathbb{E}X^* = \frac{1}{\beta}\left(\chi + \Gamma b^{H*} - \rho\sigma^2\right)$$

Since $b^G < \widehat{nfl}^{opt}$ and $\widehat{nfl}^{opt} = nfl^{opt}(\widehat{b})$, then $b^G < nfl^{opt}(\widehat{b})$.

Note that, since Conditions 1, 2 and 3 hold, as well as $b^{max} > b^G$ and $\Gamma > 0$, then, according to Lemma 1, point (ii), we have $\widehat{b} < b^{max}$. Besides, according to Lemma 1, point (iii), we have $\widehat{b} = \widehat{b}^{CBF} + b^G$. As a consequence, $\widehat{b}^{CBF} + b^G < b^{max}$, and since Conditions 1, 2 and 3 hold, then Lemma 5 applies. Lemma 5, point (i), implies that $nfl = min(b^G, nfl^{opt}(\widehat{b}))$. As a result, $nfl = min(b^G, nfl^{opt}(\widehat{b})) = b^G$, since $b^G < nfl^{opt}(\widehat{b})$.

Replacing b^{H*} in $\mathbb{E}X^*$ with $\widehat{b}$ and then nfl with b^G, we obtain

$$\mathbb{E}X^* = \frac{1}{\beta}\left(\chi + \Gamma\frac{\rho\sigma^2[1-\alpha(1+b^G)] - \chi}{\rho^2\sigma^2 + 2\Gamma} - \rho\sigma^2\right)$$

It is decreasing in σ if Γ is small, which is satisfied under Condition 3.

E.5 Fiscally-backed Interventions

We consider here an extension where we allow the central bank to take a net position. We remove the constraint on the first-period portfolio decision of the central bank (17), and we do not impose $b^{CBF} + b^{CB} = 0$. Instead, we assume

$$b^{CBF} + b^{CB} + \tau_1^{CB} = 0 \tag{E.9}$$

This means that the central bank can set b^{CB} independently from b^{CBF}. The central bank's net borrowing is $-(b^{CBF} + b^{CB})$ and is not necessarily equal to zero. A net position is financed via a government subsidy $-\tau_1^{CB}$. We therefore replace the period-1 government budget constraint (15) with

$$b^G + \tau_1^{CB} = t_1 \tag{E.10}$$

As a consequence, the equilibrium in the bond market is not directly affected by b^{CBF} as it depends on b^{CB}: $b^{H*} = b^G - b^{CB} - b^H$. As a result, the constraint $b^H \geq 0$ results in the effective constraint for the central bank

$$b^{H*} \leq b^G - b^{CB} \tag{E.11}$$

instead of (32). This implies that the central bank can change the gross foreign liabilities of the country by issuing domestic bonds (setting a more negative b^{CB}).

The effective constraint on nfl, which results from $b^F \geq 0$, remains unchanged:

$$nfl \leq b^{H*} - b^{CBF}$$

However, a FX intervention (an increase in b^{CBF}) is not necessarily offset by an increase in the gross foreign liabilities (an increase in b^{H*}), because $b^{CBF}+b^{CB}$ is not necessarily zero. To see this, note that $b^{H*}-b^{CBF}=b^{G}-(b^{CB}+b^{CBF})-b^{H}\leq b^{G}-(b^{CB}+b^{CBF})$. In fact, by increasing it net borrowing $-(b^{CB}+b^{CBF})$, the central bank can relax the constraint on the economy's net borrowing constraint nfl.

A constrained planner that performs both sterilized and fiscally-backed intervention has thus an additional choice variable, b^{CB}, and faces the constraint (E.11) instead of (32). We define the new modified constrained planner equilibrium as follows:

Definition 6 (Modified Constrained planner equilibrium) *A modified constrained planner equilibrium is an equilibrium where a planner maximizes objective* (12) *subject to the economy's resource constraints* (22)*; the asset pricing equation* (6)*; the policy rule* $f(i,S_1)=0$*; the foreign liability constraints* (E.11) *and* (33)*; and the definition of UIP* (3)*. The planner's choice variables are* $(i,S_1,b^{H*},nfl,b^{CBF},b^{CB})$.

The central bank's program in that case is:

$$
\begin{aligned}
&\max \mathbb{E}\Bigg\{ U(c_1)+\beta U(c_2)\\
&+\eta_1\left(y_1-c_1+nfl\right)\\
&+\eta_2\left[y_2-c_2-(1+i^*)nfl-\left[(1+i)\tfrac{S_1}{S_2}-(1+i^*)\right]b^{H*}\right]\\
&+\xi f(i,S_1)\\
&+\Lambda\left(b^{H*}-b^{CBF}-nfl\right)\\
&+\tilde{\Lambda}\left(b^{G}-b^{CB}-b^{H*}\right)\\
&+\alpha_0\left(\mathbb{E}\left(m^*\left[(1+i)\tfrac{S_1}{S_2}-(1+i^*)\right]\right)-\Gamma b^{H*}-\chi\right)\Bigg\}
\end{aligned}
$$

Consider the first order conditions for assets:

$$/nfl: \quad \eta_1-\mathbb{E}\left(\eta_2(1+i^*)\right) \quad -\Lambda \quad =0 \tag{E.12}$$

$$/b^{H*}: \quad -\mathbb{E}\left(\eta_2 X^*\right) \quad +\Lambda-\tilde{\Lambda}-\alpha_0\Gamma \quad =0 \tag{E.13}$$

$$/b^{CBF}: \quad -\Lambda \quad =0 \tag{E.14}$$

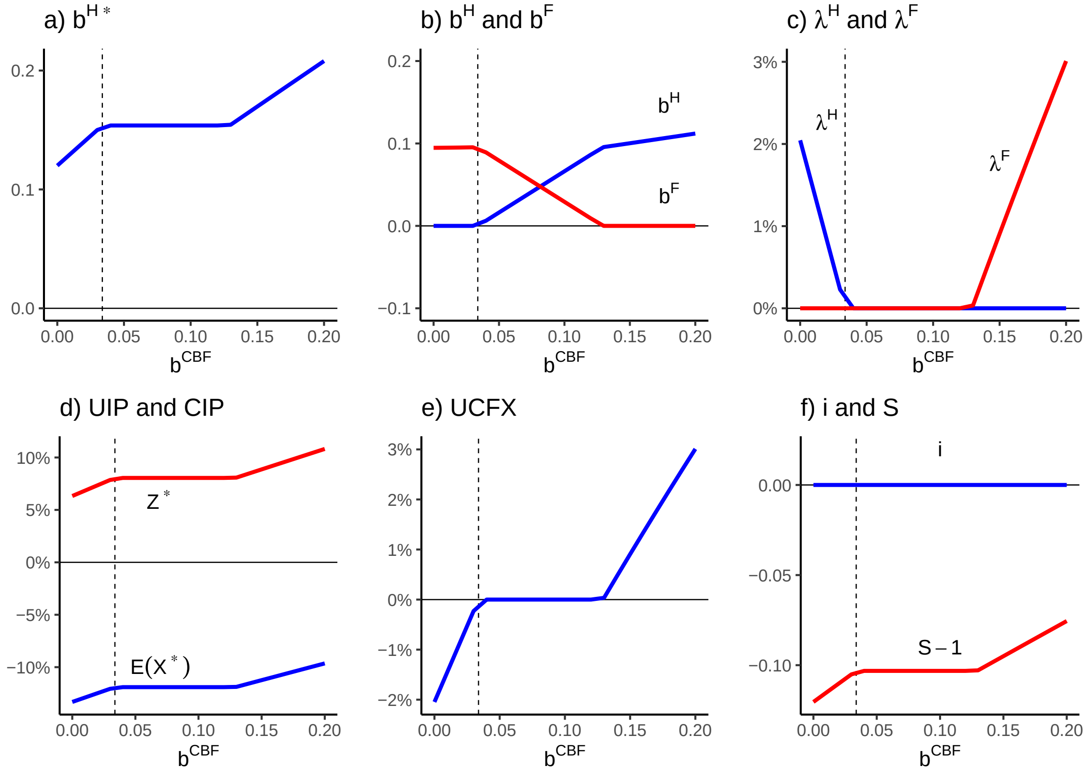
$$/b^{CB}: \quad -\tilde{\Lambda} \quad =0 \tag{E.15}$$

Equations (E.14) and (E.15) imply that $\tilde{\Lambda} = \Lambda = 0$. This means that the central bank is able to relax both its foreign-currency and domestic-currency debt constraints by adjusting its assets b^{CBF} and liabilities b^{CB}. Also, note that, as in our baseline analysis, $\eta_1 = U'(c_1)$, $\eta_2 = \beta U'(c_2)$, and that $m = \eta_2/\eta_1$ is the central bank's discount factor, which coincides with the household's following similar arguments.

As a result, Equation (E.13) implies that optimal FX intervention follow the same rule (37) as in the baseline, that is, $MBFX = 0$, and Equation (E.12) implies that intertemporal optimality is now always satisfied: $\mathbb{E}[m(1 + i^*)] = 1$.

F Additional Figures

Figure F.1: The Effectiveness of FX Interventions and the Utility Cost of Reserves - Alternative specification



Notes: Parameter values : $\beta = 0.98$, $\sigma^2 = 1$, $\chi = 0.002$, $\Gamma = 0.5$, $\alpha = 0.6$, $\rho = 0.2$, $g = 0.05$, $b^G = 0.12$. The dashed lines represent $b^{CBF} = b^{max} - b^G$.

References

Adrian, T., E. Etula, and T. Muir (2014). Financial intermediaries and the cross-section of asset returns. *The Journal of Finance 69*(6), 2557–2596.

Ahir, H., N. Bloom, and D. Furceri (2022). The world uncertainty index. Technical report, National Bureau of Economic Research.

Baker, S. R., N. Bloom, and S. J. Davis (2016). Measuring economic policy uncertainty. *The quarterly journal of economics 131*(4), 1593–1636.

Campbell, J. Y. and J. H. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of political Economy 107*(2), 205–251.

He, Z., B. Kelly, and A. Manela (2017). Intermediary asset pricing: New evidence from many asset classes. *Journal of Financial Economics 126*(1), 1–35.